Dynamic Spectrum Management for Energy-Efficient Transmission in DSL

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***Abstract*—Dynamic spectrum management (DSM) is an important technique for mitigating crosstalk noise in multi-user digital subscriber line (DSL) environments. Until now, most of the proposed algorithms for DSM have been designed solely for the purpose of bitrate maximization. These algorithms assume a fixed maximum total power and neglect the energy consumption in DSL modems. However, since recently there is a strong interest in the DSL field to reduce energy consumption as shown, *e.g.*, by the European Commissions' code of conduct on energy consumption of broadband equipment.**

In contrast to traditional DSM, this paper will show how DSM can be used for minimizing the energy consumption. We will formulate a global optimization problem for energy minimization and discuss several of its peculiarities compared to the current DSM problems. Furthermore, we derive an iterative, dual-based and semi-distributed algorithm for its local solution, which we call energy-efficient spectrum balancing (EESB). The performance of the algorithm is evaluated through simulations, which show similar results to optimal schemes. In addition, EESB achieves substantial energy savings that can be exploited by adapting the transmit powers to users' bitrate demand.

***Index Terms*—DSL, dynamic spectrum management, energy efficiency, dual relaxation.**

I. Introduction

Energy-efficiency has always been an important design criterion for wireless systems. However, it has only recently become an issue for wired communication systems. One example of how it is address is the "EU Code of Conduct on Energy Consumption of Broadband Equipment" which exists since July 2006 with the goal to half the expected electricity consumption of broadband equipment by 2015 [1]. Reducing the power consumption in wired communication systems is now also on the agenda of various standardization bodies, *e.g.* for Ethernet [2] and ETSI [3].

Up to now no systematic effort has been pursued to optimize the energy efficiency in DSL systems. The only attempt so far to reduce the energy consumption was the introduction of low-power modes in the ADSL standard, unfortunately they have largely failed due to concerns about instability in the operator's network. When a modem returns from a low-power state and transmits at full power again it will introduce time-varying crosstalk that can not be handled by currently deployed ADSL systems.

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There are of course many ways to tackle the problem of energy minimization in DSL, *e.g.*, through increasing the effectiveness of framing or implementing more appropriate congestion control mechanisms. However, in this paper we will restrict ourselves to the physical layer and even further to the problem of controlling the transmit power.

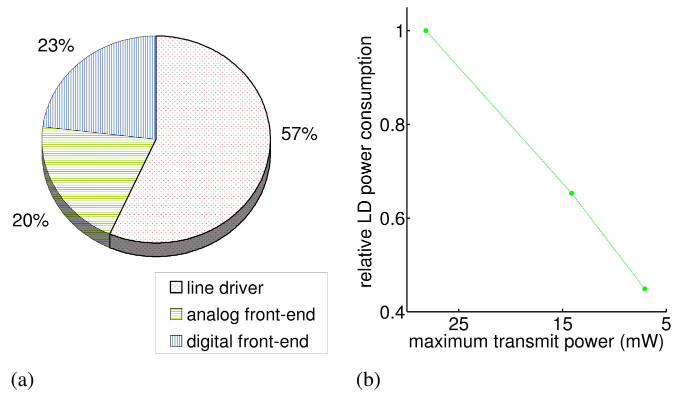

Fig. 1. a) An example of a power distribution in a current VDSL transceiver, b) dependency of normalized energy consumption in a line driver (LD) on the maximum transmit power for 17 MHz bandwidth.

From power consumption measurements of today's state-of-the-art VDSL2 chipsets we know that the line driver (LD) accounts for up to 60 %, cf. Figure 1(a). From Figure 1(b) we can conclude that a reduction in transmit power yields additional energy savings by reducing the power consumption in the LD. Thus, by optimizing the power used for transmission we can significantly lower the DSL *system* power consumption as a whole. With this potential for energy savings in mind we will introduce dynamic spectrum management (DSM) as a means to achieve energy-efficient transmission in DSL. Up to now DSM has mainly been utilized to maximize the bitrates by adapting the transmit power spectral density (PSD) of modems to the actual network environment. In recent years a number of algorithms based on dual-decomposition have been proposed for DSM which achieve optimal or near-optimal performance, *e.g.* in [4], [5], [6], [7], [8].

In this paper we formulate a global optimization problem for energy minimization and discuss several of its peculiarities compared to the current DSM problems. We will show that when convex approximations and dual decomposition are applied jointly, certain care has to be taken during algorithm design in order not to run into infeasibility problems. Furthermore, we derive an iterative, dual-based and semi-distributed local solution algorithm, which we call energy-efficient spectrum balancing (EESB). The performance of the algorithm is evaluated through simulations, which show similar results to optimal schemes. In addition EESB reveals the substantial energy savings that can be exploited by adapting the transmit power to users' bitrate demand.

The remainder of this paper is organized as follows: In Section II we present the system model and introduce the used notation. In Section III a generic DSM problem formulation for energy minimization in DSL is presented for which we derive a distributable, local optimization algorithm in Section IV. Next we exemplify our discussion in Section V through numerical simulation results obtained in a realistic VDSL scenario. Section VI finally summarizes our contributions and contains conclusions for future work.

II. System Model

We consider a DSL system consisting of U interfering lines sharing a single cable binder, where synchronized discrete multitone modulation (DMT) is employed at each modem and frequency division duplexing is used to separate transmission directions. These assumptions allow us to model the C carriers as orthogonal subchannels and neglect near-end crosstalk, yielding a far-end crosstalk limited system due to the typically comparably low background-noise levels. We assume a central unit, *e.g.* a spectrum management center at the colocated end of the cable bundle, has full knowledge of the magnitudes of crosstalk couplings and that no interference cancelation is performed. Thus modems regard crosstalk solely as noise, which for a sufficiently high number of users can be well approximated by a Gaussian distribution. Considering continuous bit-loading and two-dimensional signal constellations, the achievable rate per DMT-symbol for user $u \in \mathcal{U}$ on carrier $c \in \mathcal{C}$, where $\mathcal{U} = \{1, \ldots, U\}$ and $\mathcal{C} = \{1, \ldots, C\}$, is thus given by

$$r_c^u\left(P_c^u, \mathbf{P}_c^{\setminus u}\right) = \log_2\left(1 + \mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})\right), \tag{1}$$

where P_c^u is the power assigned to carrier c of user u, $\mathbf{P}_c^{\setminus u}$ is the vector of powers on carrier c of all users except user u and SINR_c^u denotes the corresponding signal-to-interference ratio

$$\mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u}) = \frac{H_c^{uu} P_c^u}{\Gamma(\sum\limits_{i \in \mathcal{U} \setminus u} H_c^{ui} P_c^i \; + \; N_c^u)}. \tag{2}$$

Herein H_c^{uu} and H_c^{ui} are the squared magnitudes of the direct channel transfer coefficient of user u and the cross-channel transfer coefficient from user i to user u on carrier c, respectively. Constant Γ denotes the SNR-gap to capacity depending on modulation, targeted bit-error rate, coding and noise-margin and N_c^u represents the total received background noise power on carrier c and line u, including white thermal noise, alien-crosstalk and radio-frequency interference.

III. Mathematical Program Formulation

In the following we state a mathematical program formulation for the problem of optimal DSM for power-reduction in DSL. We refer to the following as the *global optimization problem*

$$\begin{aligned} \underset{\mathbf{P} \succeq \mathbf{0}}{\text{minimize}} \quad & \sum_{u \in \mathcal{U}} w_u \sum_{c \in \mathcal{C}} \; P_c^u \\ \text{subject to} \quad & \sum_{c \in \mathcal{C}} r_c^u\left(P_c^u, \mathbf{P}_c^{\setminus u}\right) \; \geq \; R_u, \quad \forall u \in \mathcal{U}, \end{aligned} \tag{GOP}$$

where $\mathbf{P} = [(\mathbf{P}^1)^T, \ldots, (\mathbf{P}^U)^T]^T$, $\mathbf{P}^u = [P_1^u, \ldots, P_C^u]^T$ denotes the power-assignment over carriers of user u, w_u is a weighting of user u, where $w_u \geq 0$, $\sum_{u \in \mathcal{U}} w_u = 1$, and R_u is the target-rate for user u in [bits/DMT-symbol]. The total used energy per DMT-symbol can then be calculated as $1/f_s \sum_{u \in \mathcal{U}} \sum_{c \in \mathcal{C}} \; P_c^u$, where f_s denotes the DMT-symbol frequency. Therefore, assuming the power allocation and the environment does not change over time, we are minimizing the transmit-energy consumption of the system. Further note that we aim for solving a nonconvex and continuous problem, where in reality we are restricted to a discrete bit-allocation over carriers. However, a suboptimal discrete solution can be recovered, *e.g.* by rounding P_c^u, $\forall c \in \mathcal{C}$ down to power-levels associated with feasible bit-allocations [6].

We also note the similarity of (GOP) to the problem of margin-maximization in DSL [9], where however the (unweighted) objective is to maximize the (minimum) gap to capacity while still transmitting with full power. Hence the solution is considering higher crosstalk than we do in our formulation and is therefore suboptimal w.r.t. sum-power. Furthermore, we see that this formulation is complementary to the standard DSM problem formulation for rate maximization [6]. There one maximizes the weighted sum-rate subject to a fixed power budget of each user, while here the objective is to minimize the weighted total power-consumption w.r.t. a target-rate constraint for each user.

The theoretical idea behind the weighting in problem (GOP) is to allow us to trace the boundary of a "power-region". By adjusting weights we can steer the optimization to achieve (if existent!) boundary-points which additionally fulfill certain per-user total-power restrictions. This becomes possible since by decreasing a single user's weight one will never increase other users' sum-powers at a solution of (GOP). An outline of a proof of this is given in the appendix, where also the concept of a power-region is explained in more detail. Note that this idea can be seen complementary to the notion of a rate-region in the standard DSM rate maximization problem [6], where one traces its boundary to satisfy (if feasible) certain additional minimum-rate demands.

IV. Energy-Efficient Spectrum Balancing (EESB)

The first step in the derivation of our EESB algorithm for solving problem (GOP) is a convexification of the nonconvex constraints. Based on this we will later define a local optimization algorithm. An adequate approximation $\tilde{r}_c^u$, $u \in \mathcal{U}$,

$c \in \mathcal{C}$ of the corresponding rate-functions r_c^u is given by [7]

$$\tilde{r}_c^u\left(P_c^u, \mathbf{P}_c^{\setminus u}, \alpha_c^u, \beta_c^u\right) = \alpha_c^u \log_2\left(\mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})\right) + \beta_c^u, \tag{3}$$

where

$$\alpha_c^u = \mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})/(1 + \mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})) \tag{4}$$

$$\beta_c^u = \log_2\left(1 + \mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})\right) - \alpha_c^u \log_2\left(\mathrm{SINR}_c^u(P_c^u, \mathbf{P}_c^{\setminus u})\right). \tag{5}$$

Since this function underestimates the real rates, any solution of the approximated problem is also feasible w.r.t. the original one. Note however that the approximated problem turns out to be convex only after the variable transformation $\mathbf{P} = \mathrm{e}^{\tilde{\mathbf{P}}}$, see [7] for a detailed description. While the objective in (GOP) is a sum of per-user sum-powers and the positivity constraints simply constrain $\mathbf{P}$ to the (decomposable) positive orthant, it is the set of approximated constraints $\sum_{c\in\mathcal{C}} \tilde{r}_c^u(\mathbf{P}) \geq R_u,\ \forall u \in \mathcal{U}$ which couples the optimization over users. Full dual relaxation of these constraints does not change this fact, since although the problem becomes decomposable into per carrier problems similar to as shown in [6], the optimization of power-allocation on each carrier has to be performed jointly over all users. This fact motivates the application of a slightly different decomposition concept, known as optimality condition decomposition (OCD) [10]: The key idea is to form U subproblems by only doing a partial relaxation w.r.t. other-users' rate constraints, using Lagrange multipliers $\lambda_i,\ i \in \mathcal{U} \setminus u$. This allows us to spare the need of doing the update of dual variables in a centralized fashion. Furthermore, each user u only optimizes over $\mathbf{P}^u$ and assumes the primal variables $\mathbf{P}^i,\ i \in \mathcal{U} \setminus u$ and dual variables $\lambda_i,\ i \in \mathcal{U} \setminus u$ to be fixed. The u'th *user subproblem* constructed in this manner can then be written as

$$\begin{aligned}\underset{\mathbf{P}^u \succeq \mathbf{0}}{\text{minimize}}\quad & w_u \sum_{c\in\mathcal{C}} P_c^u + \sum_{i\in\mathcal{U}\setminus u} \bar{\lambda}_i \left(R_i - \tilde{r}_c^i(P_c^u, \bar{\mathbf{P}}_c^{\setminus u}, \bar{\alpha}_c^i, \bar{\beta}_c^i)\right)\\ \text{subject to}\quad & \sum_{c\in\mathcal{C}} \tilde{r}_c^u\left(P_c^u, \bar{\mathbf{P}}_c^{\setminus u}, \alpha_c^u, \beta_c^u\right) \geq R_u,\end{aligned} \tag{USP}$$

where the bars over primal and dual variables emphasize that they are kept constant during the course of a single user's optimization. The name of the applied decomposition comes from the fact that the Karush-Kuhn-Tucker (KKT)-conditions remain unchanged w.r.t. those of the original global problem (GOP). Note further that user u also considers other-user approximation-parameters $\alpha_c^i, \forall i \in \mathcal{U} \setminus u, \forall c \in \mathcal{C}$ to be fixed. In other words, we only improve the approximation of $r_c^u,\ \forall c \in \mathcal{C}$ in each step of the following iterative local optimization scheme. This is in strong contrast to [7] where both, primal *and* dual optimization aim for the optimum of an approximation which is held fixed during one sweep of a nonlinear Gauss-Seidel update [11]. For convergence reasons we also deviate from the standard OCD in [10] in that we replace the Jacobi-like update scheme [11] with non-full optimization by a nonlinear Gauss-Seidel algorithm with full per-user optimization. At this point we are free in selecting a method to optimize (USP) which gives us (*e.g.* approximately) optimal $\mathbf{P}^u$ and a Lagrange multiplier λ_u associated with the rate constraint of user u. We select standard Lagrangian relaxation to facilitate a simplifying decomposition of the problem into C per-carrier problems. Furthermore, a simple gradient-based fixed-point update of $P_c^u,\ \forall c \in \mathcal{C}$ is readily derived. Applying the above mentioned transformation $\mathbf{P} = \mathrm{e}^{\tilde{\mathbf{P}}}$, the thereby convex Lagrangian of subproblem u has the form

$$\begin{aligned}\tilde{L}^u(\tilde{\mathbf{P}}^u, \lambda_u) = &\\ & w_u \sum_{c\in\mathcal{C}} \mathrm{e}^{\tilde{\mathbf{P}}_c^u} + \sum_{i\in\mathcal{U}\setminus u} \bar{\lambda}_i \left(R_i - \sum_{c\in\mathcal{C}} \tilde{r}_c^i(\mathrm{e}^{\tilde{P}_c^u}, \bar{\mathbf{P}}_c^{\setminus u}, \bar{\alpha}_c^i, \bar{\beta}_c^i)\right)\\ & + \lambda_u(R_u - \sum_{c\in\mathcal{C}} \tilde{r}_c^u(\mathrm{e}^{\tilde{P}_c^u}, \bar{\mathbf{P}}_c^{\setminus u}, \alpha_c^u, \beta_c^u)),\end{aligned} \tag{6}$$

which notably also contains user u's rate constraint. Further derivation of $\tilde{L}^u(\tilde{P}_c^u, \lambda_u)$ w.r.t. $\tilde{P}_c^u$ where α_c^u and β_c^u are considered as constants, equation to zero and reformulation yields the fixed-point update

$$P_c^u = \frac{\lambda_u \alpha_c^u}{\log(2)\ w_u + \sum\limits_{i\in\mathcal{U}\setminus u} \frac{\bar{\lambda}_i \bar{\alpha}_c^i H_c^{iu}}{\sum\limits_{j\in\mathcal{U}\setminus\{i,u\}} H_c^{ij} \bar{P}_c^j\ +\ H_c^{iu} P_c^u\ +\ N_c^i}}. \tag{7}$$

As explained in [7] the right-hand-side in (7) is a "standard interference function" which guarantees convergence and non-negativity of P_c^u for $P_c^u \geq 0,\ \forall\ u \in \mathcal{U}$. Note however that in practice due to numerical rounding α_c^u may become zero which would prevent any further update. To circumvent this problem it is sufficient to reset α_c^u to a small positive value in case it becomes zero. We update the multiplier λ_u using a simple and efficient subgradient algorithm with adaptive step-size: On convergence of the above described local optimization scheme we obtain a subgradient $g_u(\mathbf{P}^u)$ of $\tilde{L}^u(\tilde{\mathbf{P}}^u, \lambda_u)$ w.r.t. λ_u in the form of

$$g_u(\mathbf{P}^u) = (R_u - \sum_{c\in\mathcal{C}} \tilde{r}_c^u(P_c^u, \bar{\mathbf{P}}_c^{\setminus u}, \alpha_c^u, \beta_c^u))\ |_{\mathbf{P}^u = \hat{\mathbf{P}}^u}, \tag{8}$$

where $\hat{\mathbf{P}}^u = \arg\min_{\mathbf{P}^u} \tilde{L}^u(\tilde{\mathbf{P}}^u, \lambda_u)$.

Algorithm 1 summarizes the EESB scheme, where the exact dual-update with step-size adaption is explained in detail. Therein ϵ denotes a small positive constant, Λ and Δ are initial values for λ_u and step-size *e.g.* found through some fast 1-D search as in [12], and $\{\cdot\}_+$ denotes the projection onto the positive orthant. It is important to note that the scheme from [7] is not directly applicable to problem (GOP). The above described partial update of the approximation becomes necessary since, unlike in the complementary rate maximization problem, we would otherwise update the multipliers using approximated rates – a process which could lead to heavy oscillations in dual variables or even prohibit convergence due to infeasibility of the target-rates w.r.t. the approximation, depending on the quality of the approximation

Algorithm 1 EESB Scheme

1: High-SNR initialization of the approximation:
$\alpha_c^u = 1$, $\beta_c^u = 0$, $\forall u \in \mathcal{U}, \forall c \in \mathcal{C}$
2: Initialize $P_c^u = \epsilon$, $\forall u \in \mathcal{U}, \forall c \in \mathcal{C}$
3: **repeat**
4:   **for** $u = 1$ *to* U **do**
5:     Initialize $\lambda_u = \Lambda$, $\delta = \Delta$, $\tilde{\sigma} = 0$
6:     **repeat**
7:       **repeat**
8:         Update $\mathbf{P}^u$ according to (7)
9:         Refresh α_c^u, β_c^u, $\forall c \in \mathcal{C}$ according to (4) and (5)
10:         Refresh $\tilde{r}_c^u(P_c^u, \bar{\mathbf{P}}_c^{\setminus u}, \alpha_c^u, \beta_c^u)$, $\forall c \in \mathcal{C}$ as in (3)
11:       **until** user-local primal convergence*
12:       Compute subgradient g_u according to (8)
13:       Subgradient update $\lambda_u = \{\lambda_u + \delta g_u\}_+$
14:       Step-size update $\delta =$ ***UpdateStepsize***$(g_u, \delta, \tilde{\sigma})$
15:     **until** user-local dual convergence* and feasibility
16:   **end for**
17: **until** global dual convergence*

18: ***Function*** $\delta =$ ***UpdateStepsize***$(g_u, \delta, \tilde{\sigma})$
19: Calculate $\sigma = sign(g_u)$
20: **if** $\sigma \neq \tilde{\sigma}$ **then**
21:   $\delta = \delta/2$, $\tilde{\sigma} = \sigma$
22: **end if**

* While there are various stopping-criteria in practice, for our simulations we use the change in PSD between iterations as a primal- and the change in dual-variables between per-user iterations and Gauss-Seidel sweeps as local and global dual-convergence criteria respectively.

at the current state. However, since we update the parameters of the approximation after each local optimization step in Line 9 of Algorithm 1, the multipliers are in effect updated using the real achieved rates. Contrarily, in [7] they apply the same approximation in a framework of rate maximization. There a primal/dual solution of the approximated Lagrangian meets the sum-power constraints and is hence also feasible w.r.t. the original problem. We also found that the proposed dual subgradient-update using an adaptive step-size in practice clearly outperforms simple bisection-search. This comes from the local optimization by each user of a nonconvex function $\tilde{L}^u(\tilde{\mathbf{P}}^u, \lambda_u)$, a fact not altered by the applied stepwise convexification. Bisection-search however discards half of the search-space in each iteration, which can clearly be wrong if the calculated subgradient is not the one at the primal optimum of the Lagrangian.

V. Simulation Results and Discussions

In this section we present several simulation results demonstrating our EESB algorithm, showing the potential of energy minimization in DSL and illustrating the concept of an achievable power-region from Section III. We consider a VDSL upstream scenario as depicted in Figure 2 with two users located at 300m (user 1) and 600m distance (user 2) from

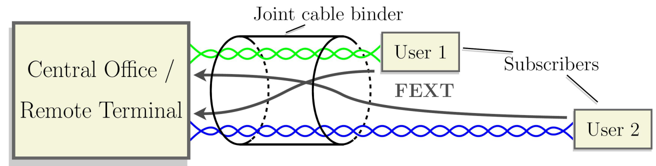

Fig. 2. VDSL upstream scenario.

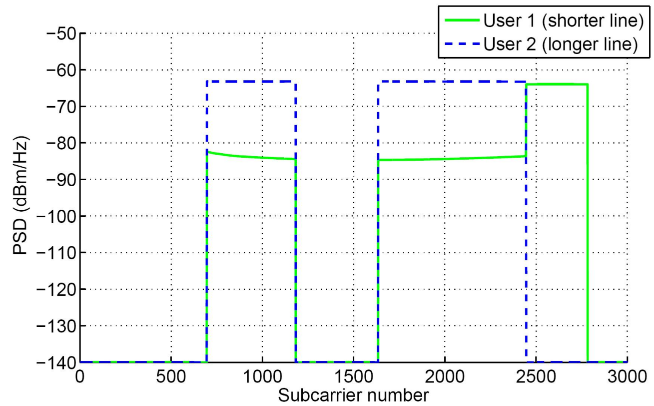

Fig. 3. Spectral power distribution for target-rates $\mathbf{R} = [45, 45]^T$ Mbps.

the deployment point, respectively. This is a near-far problem and therefore more challenging than the typical downstream situation. Furthermore, the restriction to a $2-$user scenario is only due to ease of illustration while in general EESB can deal with any number of users. The simulation parameters were set according to the ETSI VDSL standard [13]. Correspondingly we used an SNR-gap $\Gamma = 12.8$ dB and band plan 997 which defines two upstream bands. Taking alien noise into account we added ETSI VDSL Noise A to the background noise at -140 dBm/Hz. The results were computed using the sequential EESB algorithm with the sequence of user-updates starting with user 1.

In Figure 3 we show as an example the spectral power distribution obtained by EESB for target-rates $\mathbf{R} = [R_1, R_2]^T = [45, 45]^T$ Mbps. While on the longer line the power is allocated over lower frequencies, the shorter line also uses the higher ones. Intuitively this is due to the lower attenuation at low frequencies and the fact that the FEXT-crosstalk from the shorter line into the longer one is already decreasing with frequency at higher frequencies.

Figure 4 shows the minimum (unweighted) sum-power attained by EESB as a function of target-rates. Regarding the dependency of the rate-region in the complementary problem on total power, we expect this function to be convex – an intuition supported by our simulations. The exponential increase of sum-power as seen in Figure 4 reveals the dramatic potential for energy saving in DSL which could possibly be exploited through adaptation to instantaneous rate requirements, i.e., through use of cross-layer information. It can be seen that by reducing the rates by 20% from $[50, 50]$ (indicated by point *A*) to $[40, 40]$ (indicated by point *B*) the total consumed power for transmission reduces by as much as 95.9%! While here we did not distinguish between the total-power spent by each user

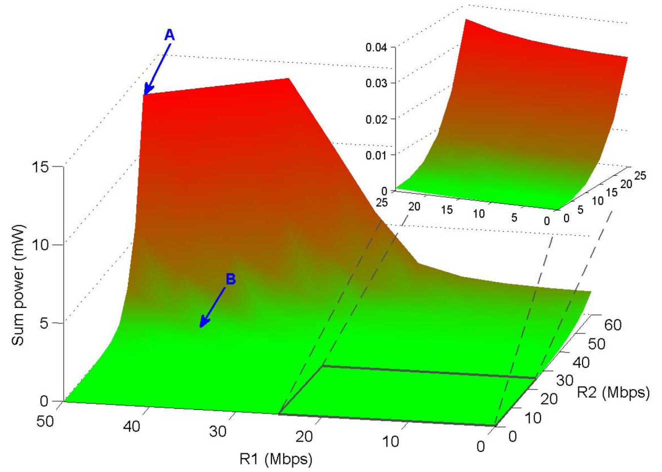

Fig. 4. Minimum sum-power achieved by EESB over target-rates, showing the energy-saving potential in DSL through rate-adaptation.

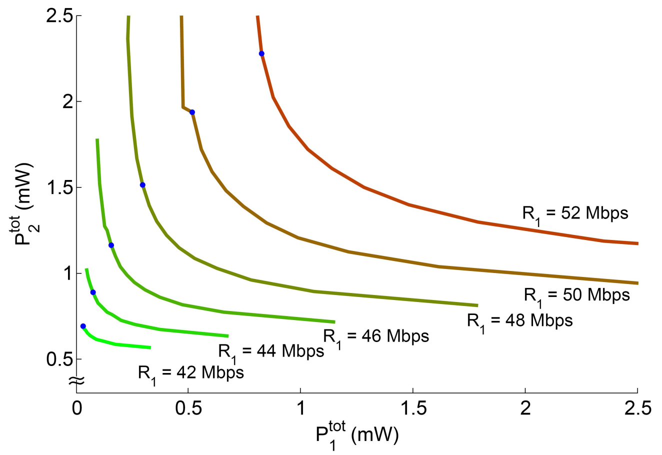

Fig. 5. Power-regions of EESB for target-rates $R_2 = 40$Mbps and R_1 between 42Mbps and 52 Mbps (sum-powers P_1^{tot}, P_2^{tot} of user 1 and 2 respectively; marked points resulted from equal weights for each user).

separately, we plot in Figure 5 the power-regions of EESB for target-rates constant for user 2 at 40 Mbps and increasing for user 1 in steps of 2 Mbps from 42 Mbps up to 52 Mbps. As expected from Section III the curves are approximately convex for lower target-rates and most choices of weights, and their distance to each other seems to increase exponentially. However, we observed that for increasing target-rates and unbalanced weighting it becomes more and more challenging to achieve meaningful curves due to the increasing sensitivity of primal solutions on changes of Lagrange multipliers. Note that taking the point of each curve obtained by applying equal weights (cf. Figure 5) we have the points in Figure 4 for the corresponding target-rates. Theoretically the points according to equal weights are sum-power optimal. In Figure 5 though we recognize that EESB implicitly puts preference on user 1 for the chosen update-sequence, yielding shifted equal-weight points. We see that still the weights in (GOP) allow one to privilege certain users. However, this strategy seems to become increasingly hard to implement for higher target-rates due to

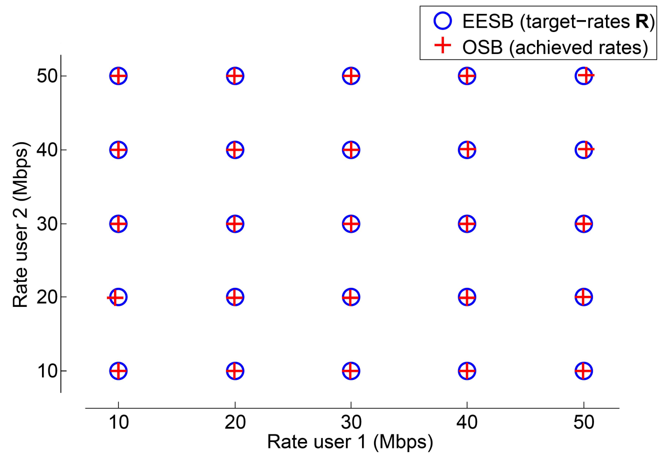

Fig. 6. Comparison to a "guided" optimal spectrum balancing (OSB) algorithm, showing the small sub-optimality of EESB ($< 0.5\%$ in rate/user).

the increasing nonconvexity of the problem.

The suboptimality of iterative schemes such as iterative spectrum balancing (ISB) [4], [5] in comparison to optimal spectrum balancing (OSB) [6] is well-known [5]. To investigate the suboptimality of EESB, in the following we compare it to the original OSB-algorithm which uses a discrete grid-search over bit-allocations per user and carrier. We guide OSB by setting the maximum sum-power constraints according to the solution obtained by EESB for target-rates $\tilde{\mathbf{R}} = [\tilde{R}_1, \tilde{R}_2]^T$. Next we search the boundary of the rate-region for a point $\mathbf{R}$ achieving the rate-relation $\tilde{R}_1/\tilde{R}_2$. Given that at this point the maximum sum-power is used we allow us to compare the thereby achieved rates $\mathbf{R}$ to the above target-rates $\tilde{\mathbf{R}}$. The thereby obtained points are plotted in Figure 6, where we see that for most target-rate combinations OSB obtains solutions very similar to our target-rates. More precisely, the suboptimality of EESB in terms of per-user rates was found to be less than 0.5% at the examined target-rate combinations. Comparing the two different problems however it is obvious that one would have to exhaustively search for all possible maximum sum-power combinations in order to get a picture of the OSB-optimal solution. Although EESB performs sub-optimally, as seen in Figure 5, we find that EESB's target rates are almost identical to the OSB solution when OSB is guided to use the sum-powers found by EESB.

VI. Conclusions

In this paper we have formulated a global optimization problem (GOP) aiming at minimizing total energy consumption in DSL while still adhering to certain rate requirements. We have proposed an energy-efficient spectrum balancing (EESB) algorithm for solving this GOP, which has specifically been developed to cope with practical convergence issues emerging in realistic scenarios. It relies on a decomposition of the problem among users with respect to primal *and* dual updates. Furthermore, EESB allows for a semi-distributed implementation.

Comparing EESB to a "guided" optimal spectrum balancing (OSB) solution, we find the solutions similar. We have

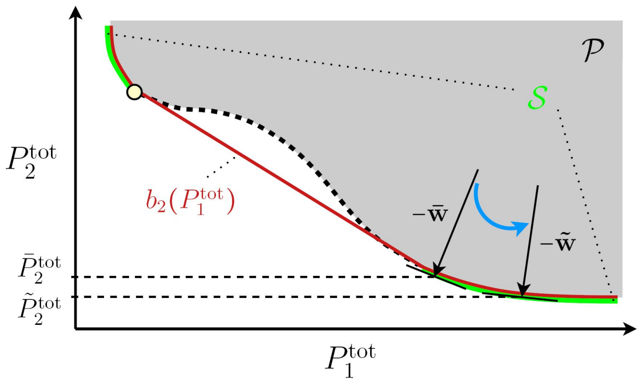

Fig. 7. Schematic illustration of the "power-region" $\mathcal{P}$, the optimal achievable set $\mathcal{S} \subseteq \text{bd}(\mathcal{P})$ and the influence of weights $\mathbf{w}$ on users' sum-powers $\mathbf{P}^{\text{tot}}$.

furthermore shown through simulations that EESB is able to achieve substantial energy savings by adapting to varying user demands. Future work in this line of research includes investigating other optimization techniques going beyond pure dual decomposition and finding ways to make dual-based DSM algorithms applicable in practice.

APPENDIX

To render the idea of a "power-region" introduced in Section III more precisely let $P_u^{\text{tot}} = \sum_{c \in \mathcal{C}} P_c^u$ and denote $\mathbf{P}^{\text{tot}} = [P_1^{\text{tot}}, \ldots, P_U^{\text{tot}}]^T$. The achievable power-region $\mathcal{P}$ of the system for given target-rates can then be defined as

$$\mathcal{P} = \{\mathbf{P}^{\text{tot}} \,|\, \sum_{k \in \mathcal{C}} r_k^u \left(P_k^u, \mathbf{P}_k^{\setminus u}\right) \geq R_u, \quad P_c^u \geq 0, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}\}. \tag{9}$$

Due to the nonconvexity of the full Lagrangian of our problem, $\mathcal{P}$ is in general nonconvex and we are not able to find every point on the boundary of $\mathcal{P}$, bd$(\mathcal{P})$, by changing the weights $\mathbf{w} = [w_1, \ldots, w_U]^T$ in (GOP), cf. Figure 7. More precise, the set of achievable boundary points $\mathcal{S}$ can be described as

$$\mathcal{S} = \mathcal{P} \cap \text{bd}(\text{conv}(\mathcal{P})), \tag{10}$$

where conv$(\cdot)$ denotes the convex hull. We also know that in the extreme cases where all weights are zero except that of user u we get kind of a "waterfilling solution" for user u where all other users allocate any power to reach their target-rates with minimal disturbance to user u. Interpreting conv$(\mathcal{P})$ as the epigraph of one of the U possible $U-1$ dimensional functions $b_u(P_i^{\text{tot}},\ \forall i \in \mathcal{U} \setminus u) = P_u^{\text{tot}}$ defined over $\mathcal{S}$, where $u \in \mathcal{U}$, we can conclude that $b_u(P_i^{\text{tot}},\ \forall i \in \mathcal{U} \setminus u)$ is in general noncontinuous and monotonously decreasing. Hence, making a step from a point $\bar{\mathbf{P}}^{\text{tot}} \in \mathcal{S}$ to a point $\tilde{\mathbf{P}}^{\text{tot}} \in \mathcal{S}$ by increasing $\bar{P}_u^{\text{tot}}$ of a single user u will not increase $\bar{P}_i^{\text{tot}},\ i \in \mathcal{U} \setminus u$. By looking at the KKT-conditions of a modified problem of (GOP) where we assume an additional sum-power constraint $\sum_{c \in \mathcal{C}} P_c^u \geq \tilde{P}_u^{\text{tot}}$ and a multiplier $\nu \geq 0$ associated with it, we see that the increase of user u's total-power is equivalent to a decrease of w_u by ν^*, where ν^* is the optimal Lagrange multiplier. Hence we see that decreasing a single user's weight will never increase $P_i^{\text{tot}}, i \in \mathcal{U} \setminus u$ at a solution of (GOP). Figure 7 illustrates this idea, where we have $\bar{\mathbf{w}} = [\bar{w}_1, \bar{w}_2]^T, \tilde{\mathbf{w}} = [\tilde{w}_1, \bar{w}_2]^T,\ \tilde{w}_1 < \bar{w}_1$, and hence for the associated optimal points $\bar{\mathbf{P}}^{\text{tot}}, \tilde{\mathbf{P}}^{\text{tot}} \in \mathcal{S}$ it necessarily holds that $\tilde{P}_2^{\text{tot}} \leq \bar{P}_2^{\text{tot}}$.

Several results in the optimization-literature show that the duality gap, and hence the afore mentioned nonconvexity of $\mathcal{P}$ vanishes as the number of subproblems (i.e., carriers) goes to infinity. In practice however it can be observed that $\mathcal{P}$ is approximately convex for a sufficiently high but finite number of carriers (see Figure 5). This expectation is further supported by corresponding results reported for OFDMA systems, *e.g.* in [14]. However, even if we assume $\mathcal{P}$ to be convex the weights do not lose their meaning. By applying equal weights we still may find optimal points $\mathbf{P}^{\text{tot}}$ which are infeasible w.r.t. additional per-user total-power constraints while such a feasible solution may exist in $\mathcal{S}$.

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