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Delay-Constrained Scheduling for Interference-Limited Multi-Carrier Systems

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***Abstract*—The reduction of energy consumption in digital subscriber line (DSL) networks has obtained considerable attention recently. Today's DSL is designed under an "always on" principle to keep the crosstalk noise as stable as possible. Departuring from this restriction, one approach to achieve energy savings is by "lazy scheduling" which exploits the tradeoff between energy-consumption and transmission delay inherent in many communication systems.**

This work extends the scope of this idea to multi-user interference limited systems employing multi-carrier modulation. Mathematical decomposition appears to be a natural approach for cross-layer optimization when the physical-layer spectrum management algorithm is already based on dual relaxation. We identify Benders decomposition as the appropriate choice of an optimization scheme for rate and delay constrained energy-minimization. Based on this we propose a cross-layer scheduler for multi-user/multi-carrier systems. By simulations of a single-hop, multi-user DSL scenario this scheduler is shown to closely approximate the optimal solution to this nonconvex problem. Furthermore, by example we demonstrate that scheduling for interference avoidance in DSL yields negligible additional performance gains over sole physical layer spectrum balancing in practice.

I. Introduction

In this work we study the delay aware energy-minimization problem by means of cross-layer scheduling in a multi-user interference-limited digital subscriber line (DSL) network employing discrete multi-tone (DMT) modulation. In single-link communication the common term "lazy scheduling" refers to the idea of lowering transmit energy consumption by lowering transmission rates and hence increasing the transmission delay [1]. Reducing rates in discrete multi-tone (DMT) systems has recently been shown to effectively reduce the transmit energy [2]. However, transmission time cannot be arbitrarily increased since data packets may have stringent delay constraints.

This work extends the scope of energy-efficient scheduling in two dimensions, *cf.* Figure 1(a). On the one hand we regard *multi-carrier* systems, where Lagrange multipliers associated with the spectrum management problem will have the interpretation of marginal costs, *cf.* scheduling without frequency diversity [3], [4]. Second, the more general case of a *multi-user* interference channel will be considered. We will present a near-optimal algorithm based on generalized

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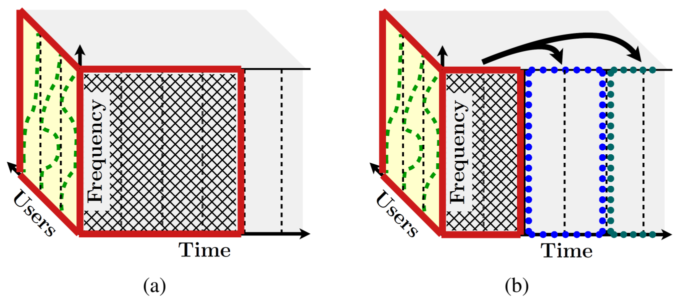

Fig. 1. Considered Scheduling Problems; 1(a) Finite-Horizon Lazy Scheduling; 1(b) Periodic Scheduling for Interference Avoidance.

Benders decomposition [5] for optimizing this generic and in general nonconvex scheduling problem. Additionally note that this scheduler can straightforwardly be adapted for wireless multi-carrier systems as well.

Our problem is related to that studied in [6] in the context of cross-layer optimization by network utility maximization for wireless networks. Therein, the nonlinear column generation method resulted in separate physical-layer and scheduling problems, where the latter employs time-divisioning to alleviate interference and improve utility. This scheduler however would not achieve any performance gain when applied to our problem as we consider linear energy cost functions. While not exploiting traffic variability, the idea of constructing schedules that are periodically repeated in time is still appealing due to lower complexity and feasible avoidance of delay violations, *cf.* Figure 1(b).

In this paper we will study, by simulation, the performance gain by periodic scheduling for interference avoidance. Theoretically our scheduling algorithm will allow for energy reduction even in case of linear objectives. It can be expected, however, that the energy-savings achieved by solving this problem are fairly small in practical DSL networks. This is partly motivated by previous results for a simplistic model with two users and two carriers in [7].

This work is structured as follows. In Section II we present our system model and introduce a program formulation for multi-user finite-horizon scheduling in DMT-based systems. Next, in Section III the Benders decomposition for energy-efficient multi-carrier scheduling (BEEMS) algorithm is proposed to alleviate complexity by modularization into a scheduling and a spectrum management problem. Comments on the sub-optimality of the approach in the multi-user case and issues related to infeasibility are given separately in Section III-A. The convergence of BEEMS will be demonstrated by simulation in Section IV, where we quantify the performance gain by periodic scheduling for interference avoidance in DSL. After summarizing our contributions in Section V we finally outline directions of future work.

II. Model, Notation and Problem Formulation

In the following we will adopt the physical layer system model and related assumptions from [8]. Therein we have a DSL multi-user system limited to spectral level cooperation and employing discrete multi-tone (DMT) modulation with C orthogonal and static subchannels. We associate users and carriers with the sets of indices $\mathcal{U} = \{1, \ldots, U\}$ and $\mathcal{C} = \{1, \ldots, C\}$, respectively. Furthermore, we denote the rate per DMT-frame on carrier c of user u as

$$r_c^u\left(\mathbf{p}_c^{(n)}\right) = \log_2\left(1 + \frac{H_c^{uu} p_c^{u,(n)}}{\Gamma(\sum\limits_{i \in \mathcal{U} \setminus u} H_c^{ui} p_c^{i,(n)} + N_c^u)}\right), \quad (1)$$

where $\mathbf{p}_c^{(n)} = [p_c^{1,(n)}, \ldots, p_c^{U,(n)}]^T$, $p_c^{u,(n)}$ is the power spectral density of user u on carrier c in frame n and we will compactly capture the power-allocation in frame n by $\mathbf{p}^{(n)} = [(\mathbf{p}_1^{(n)})^T, \ldots, (\mathbf{p}_C^{(n)})^T]^T$. The squared magnitudes of the direct channel transfer coefficient of user u on carrier c are written as H_c^{uu} and of the cross-channel transfer coefficient from user i to user u as H_c^{ui}, respectively. The SNR-gap to capacity [9] is denoted by Γ and N_c^u symbolizes the total background noise power spectral density on carrier c and line u.

For the scheduling part we adopt a fluid packet departure model, assuming knowledge of all packet arrivals in a finite scheduling horizon. Hence, packets can arbitrarily be split and transmitted so that all data arrive before the packets' deadline. Scheduling of multi-carrier transmissions is naturally slotted in multiples of the inverse symbol frequency, and the data arriving during one symbol frame cannot be scheduled before the next frame. For simplicity of notation we assume in the forthcoming a single flow per user $u \in \mathcal{U}$ and an associated packet-individual maximum delay $D_u^{\max}$, equal for each packet of user u. The size of all S data-packets, be it in the backlog or arriving in the scheduling horizon, are captured by a vector $\mathbf{s} \in \mathcal{R}_+^S$. The frames in the scheduling horizon of length N are indexed by $\mathcal{N} = \{1, \ldots, N\}$. Furthermore, we denote the vector of scheduling variables by $\mathbf{d} = [(\mathbf{d}^1)^T, \ldots, (\mathbf{d}^U)^T]^T$, where $\mathbf{d}^u = [(\mathbf{d}^{u,(1)})^T, \ldots, (\mathbf{d}^{u,(N)})^T]^T$ and $\mathbf{d}^{u,(n)} = [d_1^{u,(n)}, \ldots, d_{D_u^{\max}}^{u,(n)}]^T$ is the vector of the number of bits with remaining maximum delay between 1 and $D_u^{\max}$ frames, assigned by user u to frame n. It is this vector that defines when and which data are sent, where data transmissions at different time frames associated with the same packet are modeled by different variables. Similar to a node-incidence matrix on graphs, we define a binary block-diagonal matrix $\mathbf{M}$ of dimension $S \times (N \sum_{u \in \mathcal{U}} D_u^{\max})$ which maps between the scheduling variables and the packet-data they serve. More precisely, each row associates a packet to all the (up to $D_u^{\max}$) scheduling variables which serve a packet of user u. We assume a delay cost vector $\mathbf{c}$ of the same dimensions as $\mathbf{d}$, associated with having a single bit waiting a discrete amount of time up to its deadline. The entries may be regarded as the values of various *arbitrary* (*e.g.*, sigmoidal [10]) flow or packet specific delay-cost functions, sampled at multiples of the frame-length. This delay cost can be seen to serve two purposes. On the one hand it will allow for a variable tradeoff between energy and delay, while on the other hand it models a first-come-first-serve packet-scheduling policy, where packets with higher delay costs are preferably scheduled. Furthermore, we note that users/flows may have different costs and delay-constraints, as modeled by $\mathbf{c}$ and $\mathbf{M}$.

We formulate a generic cross-layer problem allowing for an energy-delay tradeoff as

$$\begin{aligned}
\underset{\mathbf{d} \succeq \mathbf{0},\ \mathbf{p}^{(n)}, \forall n \in \mathcal{N}}{\text{minimize}} \quad & \theta \sum_{n \in \mathcal{N}, c \in \mathcal{C}} \mathbf{w}^T \mathbf{p}_c^{(n)} + (1-\theta)\ \mathbf{c}^T \mathbf{d} && (2a)\\
\text{subject to} \quad & \mathbf{p}_c^{(n)} \in \mathcal{P}_c, \quad \forall n \in \mathcal{N}, \forall c \in \mathcal{C}, && (2b)\\
& \sum_{c \in \mathcal{C}} r_c^u\left(\mathbf{p}_c^{(n)}\right) \geq \mathbf{1}^T \mathbf{d}^{u,(n)}, && \\
& \qquad \forall n \in \mathcal{N}, \forall u \in \mathcal{U}, && (2c)\\
& \mathbf{M}\mathbf{d} \succeq \mathbf{s}. && (2d)
\end{aligned}$$

Therein we denote the tradeoff coefficient by $\theta \in [0, 1]$ and user weights by $\mathbf{w} \in \mathcal{R}_+^U$, where $\sum_{u \in \mathcal{U}} w_u = 1$. The objective (2a) is simply a weighted sum of power-expenditure (proportional to energy-consumption) and the total delay cost. Constraints (2b) and (2c) denote the feasible power-allocations, where $\mathcal{P}_c = \{\mathbf{p}_c^{(n)} \mid p_c^{u,(n)} \geq 0, r_c^u\left(\mathbf{p}_c^{(n)}\right) \in \mathcal{B}\}$, $\mathcal{B} = \{0, 1, \ldots, B^{\max}\}$ is the discrete set of feasible bit-allocations per carrier and we assume a maximum number of loaded bits per carrier $B^{\max}$. Constraint (2c) enforces that the total number of bits scheduled for a frame are supported by the physical layer spectrum management. It is the discreteness of $\mathcal{B}$ and the nonconcavity of $r_c^u\left(\mathbf{p}_c^{(n)}\right)$ in the multi-user case which make this problem nonconvex. The data-conservation constraint (2d) finally assures that all data are scheduled.

Note that this model can easily be extended to the case of multiple flows by thinking of them as multiple "users", where only the rate constraint in (2c) has to be adapted accordingly.

III. Benders decomposition for energy-efficient multi-carrier scheduling (BEEMS)

In the following we approach the optimization of the global scheduling problem (2) by modularization using generalized Benders decomposition ("constraint generation") [5]. This method is commonly applied in case of coupling variables, *cf.* variables $\mathbf{d}$ in (2). By inspection of Problem (2) we recognize that it consists of two subproblems which are coupled by the target-rate constraints in (2c). The first is a spectrum management subproblem in each frame n, given as

$$P^{(n)}(\mathbf{d}) = \underset{\mathbf{p}^{(n)}}{\text{minimize}} \quad \sum_{c\in\mathcal{C}} \mathbf{w}^T \mathbf{p}_c^{(n)} \tag{3}$$
$$\text{subject to} \quad \mathbf{p}_c^{(n)} \in \mathcal{P}_c, \quad \forall c \in \mathcal{C},$$
$$\sum_{c\in\mathcal{C}} r_c^u \left(\mathbf{p}_c^{(n)}\right) \geq R_u^{(n)}, \ \forall u \in \mathcal{U},$$

where the target rates are given by $R_u^{(n)} = \mathbf{1}^T \mathbf{d}^{u,(n)}$. In our derivation we replace (3) by a dual subproblem (DSP$^{(n)}$). Denoting the Lagrangian term of user u on carrier c in frame n by $L_c^{u,(n)} = w_u p_c^{u,(n)} + \lambda_u^{(n)}(R_u^{(n)}/C - r_c^u(\mathbf{p}_c^{(n)}))$ using multipliers $\boldsymbol{\lambda}^{(n)} \in \mathcal{R}^U$, it is given as

$$\underset{\boldsymbol{\lambda}^{(n)}\succeq\mathbf{0}}{\text{maximize}} \ \underset{\mathbf{p}^{(n)}}{\text{minimize}} \quad \sum_{c\in\mathcal{C},u\in\mathcal{U}} L_c^{u,(n)}$$
$$\text{subject to} \quad \mathbf{p}_c^{(n)} \in \mathcal{P}_c, \quad \forall c \in \mathcal{C}. \tag{DSP$^{(n)}$}$$

We refer to its optimal objective as $D^{(n)}(\mathbf{d})$ and denote the weighted power at optimum by $E^{(n)}(\mathbf{d}) = \sum_{c\in\mathcal{C}} \mathbf{w}^T \mathbf{p}_c^{(n)}$. While we note that due to nonconvexity Problems (3) and (DSP$^{(n)}$) are in general not equivalent, the *optimum* of the latter has the advantage of being convex [11, Ch. 5] (but not continuously differentiable) in the scheduling variables $\mathbf{d}$, a prerequisite for applying the generalized Benders decomposition. Note however that the minimization over $\mathbf{p}^{(n)}$ in (DSP$^{(n)}$) is still a nonconvex problem. We will discuss this approximation step in more detail in Section III-A. The second part is a scheduling master problem (SMP) which in epigraph form [12, p. 134] can be written as

$$\underset{\mathbf{d}\succeq\mathbf{0},\ t}{\text{minimize}} \quad t \tag{SMP}$$
$$\text{subject to} \quad \theta \sum_{n\in\mathcal{N}} D^{(n)}(\mathbf{d}) \ + \ (1-\theta)\ \mathbf{c}^T\mathbf{d} \leq t,$$
$$\mathbf{M}\mathbf{d} \succeq \mathbf{s}.$$

The decomposition now suggests an iterative procedure that consists of a subproblem stage in which we optimize the spectrum balancing problem (DSP$^{(n)}$) for each frame separately given a fixed bit-allocation $\mathbf{d}$. For the solution of these subproblems we refer to the dual-based energy-efficient optimal spectrum balancing algorithm in [8]. After each iteration we hence obtain the optimized total dual cost $\sum_{n\in\mathcal{N}} D^{(n)}(\mathbf{d})$ and a subgradient with respect to $\mathbf{d}$ as given by the following proposition.

Proposition 1: Assume a binary matrix $\mathbf{T}$ of dimension $(N\sum_{u\in\mathcal{U}} D_u^{\max}) \times (UN)$ mapping from the stacked vector of multipliers $\boldsymbol{\lambda}^{(n)}$, $\forall n \in \mathcal{N}$, to the scheduling variables $\mathbf{d}$ according to the association of the variables to frames and users. Then a subgradient of $\sum_{n\in\mathcal{N}} D^{(n)}(\mathbf{d})$ with respect to $\mathbf{d}$ is given as

$$\mathbf{g}(\mathbf{d}) = \mathbf{T}\left[(\boldsymbol{\lambda}^{(1)})^T, \ldots, (\boldsymbol{\lambda}^{(N)})^T\right]^T. \tag{4}$$

The complete proof is omitted here due to space restrictions. However, this proposition can easily be shown by invoking Danskin's theorem [11, p. 737]. The possible non-uniqueness of the subgradient and hence the necessity of a subgradient based optimization method follows from integer-bitloading and the hence polyhedral convex hull of the sets $\mathcal{X}^{(n)} = \sum_{c\in\mathcal{C}} \mathcal{X}^{c,(n)}$, $n \in \mathcal{N}$, where

$$\mathcal{X}^{c,(n)} = \{\mathbf{x}^c | \mathbf{x}^c = [\mathbf{r}_c\left(\mathbf{p}_c^{(n)}\right), \mathbf{w}^T\mathbf{p}_c^{(n)}]^T, \mathbf{p}_c^{(n)} \in \mathcal{P}_c\}, \tag{5}$$

and $\mathbf{r}_c(\mathbf{p}_c^{(n)}) \in \mathcal{R}_+^U$ is a vector-valued function with the u'th element given by $r_c^u(\mathbf{p}_c^{(n)})$, *cf.* also [11, ch. 5].

In order to exploit the collected subgradient information fully we construct a polyhedral outer approximation of the first constraint in (SMP), given by

$$\mathbf{A}\begin{bmatrix}\mathbf{d}\\ t\end{bmatrix} + \mathbf{f} \preceq \mathbf{0}. \tag{6}$$

Therein $\mathbf{f} \in \mathcal{R}^{2k}$, k denoting the scheduling iteration, where corresponding to a schedule $\mathbf{d}$ in iteration i we have the $2i-1$'th and $2i$'th element of $\mathbf{f}$ given as

$$f_{2i-1} = f_{2i} = \theta \sum_{n\in\mathcal{N}} D^{(n)}(\mathbf{d}) \ + \ (1-\theta)\ \mathbf{c}^T\mathbf{d}, \tag{7}$$

and the corresponding rows in $\mathbf{A} \in \mathcal{R}^{2k\times(N\sum_{u\in\mathcal{U}} D_u^{\max})+1}$ as

$$\left(\mathbf{a}^{2i}\right)^T = \left[\ (\theta\ \mathbf{g}(\mathbf{d}) + (1-\theta)\,\mathbf{c})^T \ \middle|\ -1\ \right], \tag{8}$$
$$\left(\mathbf{a}^{2i-1}\right)^T = [0, \ldots, 0, 1]. \tag{9}$$

The second stage of the scheduler hence first involves updating its approximation according to new constraints as described in (7) – (9), and then updating the scheduling variables. While the latter update can be performed in numerous ways [13], [14], we rely on the practically efficient analytic center cutting plane method [13]. Hence, at iteration k instead of (SMP) we solve the convex optimization problem

$$\underset{\mathbf{y},\mathbf{d}\succeq\mathbf{0},t}{\text{minimize}} \quad -\sum_{i=1}^{2k} \log(y_i) \tag{10}$$
$$\text{subject to} \quad \mathbf{y} = -\mathbf{f} - \mathbf{A}\begin{bmatrix}\mathbf{d}\\ t\end{bmatrix}, \quad \mathbf{M}\mathbf{d} \succeq \mathbf{s}.$$

Subsequently we have a new set of target-rates $R_u^{(n)}, \forall u \in \mathcal{U}, n \in \mathcal{N}$, from where the procedure continues as described.

With respect to the initialization of the search region for $\mathbf{d}$ we note that the maximum bit-loading constraints in (DSP$^{(n)}$) allow to efficiently search the optimum but also should be taken into consideration by the scheduler. This gives us initial cuts according to $d_i^{u,(n)} \leq B^{\max} \cdot C$. Another source of initial constraints is the maximum number of bits that can possibly be scheduled per frame based on the available data.

A basic description of the BEEMS algorithm is given in Algorithm 1. As we note that the spectrum management problems are in fact almost identical, differing only in the target-rate assignment, one might immediately extend Algorithm 1 to incorporate multiple cuts in each iteration. An intelligent

Algorithm 1 BEEMS Algorithm

Initialize target-accuracy ε, the best found primal objective $E=\infty$, and the cuts in $\mathbf{A}$ and $\mathbf{f}$, *cf.* Equation (6)

repeat

  Solve (10) to obtain $\mathbf{d}$

  for $\forall n \in \mathcal{N}$ **do**

    Solve (DSP$^{(n)}$) to obtain the optimal dual objective $D^{(n)}(\mathbf{d})$ and the corresponding weighted energy term $E^{(n)}(\mathbf{d}) = \sum_{c\in\mathcal{C}} \mathbf{w}^T \mathbf{p}_c^{(n)}$

  end for

  Update $E = \min\{E, \theta \sum_{n\in\mathcal{N}} E^{(n)}(\mathbf{d}) + (1-\theta)\ \mathbf{c}^T\mathbf{d}\}$

  Evaluate subgradient $\mathbf{g}(\mathbf{d})$ according to (4)

  Update the approximation as in Equations (7) – (9)

  Solve (11) to obtain the lower objective bound η

until $\left(\frac{E-\eta}{E} \leq \varepsilon\right)$ or convergence of $\mathbf{d}$

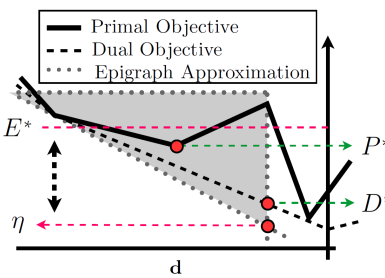

Fig. 2. Schematic Illustration of the Suboptimality-bound, including Optimal Primal, Dual and Approximated Objectives P^*, D^* and η, and the Best Primal Objective E^* Obtained by Dual Optimization in (DSP$^{(n)}$), respectively.

selection-algorithm using only a few of the "best" cuts is however out of the scope of this work.

A. *Suboptimality and Infeasibility*

In the following we discuss two issues we overlooked sofar, namely suboptimality and infeasibility. First we justify the departure from the original, primal subproblem (3) motivated by the need for a convex objective in the applied decomposition. While we note that Problems (3) and (DSP$^{(n)}$) are equivalent in the single-user case under relaxation of the integer bit-loading constraints and assuming strict feasibility of (3) [12, p. 226], in general we have that $P^{(n)}(\mathbf{d}) - D^{(n)}(\mathbf{d}) \geq 0$, *i.e.*, the optimal objective in (DSP$^{(n)}$) is a convex underestimator of the original one. Recent results however show that the gap may be negligible in practical multi-carrier systems, depending on the target-rates [8]. Another way to look at it is that applying the decomposition scheme directly on the primal subproblems (3) one would work with approximated subgradient information. A general bound on the suboptimality of this subgradient information seems however intractable. Considering the dual optimum $D^{(n)}(\mathbf{d})$ as our objective we deal with *exact* subgradient information, and further are able to quantify the suboptimality of this approach, *e.g.*, by weak duality [12, p. 225].

An important feature of the applied cutting plane method is that one has a lower bound η on the globally optimal dual objective which is computable by solving the linear program

$$\eta = \underset{\mathbf{d}\succeq\mathbf{0},\ t}{\text{minimize}} \quad t \tag{11}$$
$$\text{subject to} \quad \mathbf{A}\begin{bmatrix}\mathbf{d}\\ t\end{bmatrix} + \mathbf{f} \preceq \mathbf{0}, \quad \mathbf{Md} \succeq \mathbf{s}.$$

Hence, denoting $\mathcal{D} = \{\mathbf{d}|\mathbf{Md} \succeq \mathbf{s}, \mathbf{d} \succeq \mathbf{0}\}$ we may infer (*cf.* Figure 2)

$$\eta \ \leq\ D^* \ =\ \min_{\mathbf{d}\in\mathcal{D}} \theta \sum_{n\in\mathcal{N}} D^{(n)}(\mathbf{d}) \ +\ (1-\theta)\ \mathbf{c}^T\mathbf{d} \tag{12a}$$
$$\leq\ P^* \ =\ \min_{\mathbf{d}\in\mathcal{D}} \theta \sum_{n\in\mathcal{N}} P^{(n)}(\mathbf{d}) \ +\ (1-\theta)\ \mathbf{c}^T\mathbf{d} \tag{12b}$$
$$\leq\ E^* \ =\ \min_{\mathbf{d}\in\mathcal{D}} \theta \sum_{n\in\mathcal{N}} E^{(n)}(\mathbf{d}) \ +\ (1-\theta)\ \mathbf{c}^T\mathbf{d}, \tag{12c}$$

where the first, second and third inequality follow from the outer approximation, weak duality, and the definition of $E^{(n)}$, respectively. In Algorithm 1 this bound is used as a stopping-criterion.

A second point is the possible infeasibility of the dual subproblems (DSP$^{(n)}$). While infeasibility can be neatly incorporated if the epigraph-constraints were given in functional form [14], here the constraints are only evaluated implicitly by solving the spectrum management subproblems. A way to obtain a value for $D^{(n)}, \forall n \in \mathcal{N}$, and $\mathbf{g}(\mathbf{d})$ even in case of infeasibility is by turning (DSP$^{(n)}$) into an always feasible problem, *cf.* [15, Sec. 3.3]. While not used in our simulations, we may also make the bit allocation always feasible by artificially limiting the constraint-set in the master problem (SMP). This can be done, *e.g.*, by incorporating the optimum rates from a rate-maximization spectrum management problem, constrained by the maximum bit-loading. Note that while a convergence proof is available for the used cutting plane method [13], it does not apply in our case due to the mentioned infeasibility issue.

IV. Simulations on Periodic Scheduling

We now demonstrate the convergence of the proposed BEEMS algorithm by applying it to the problem of computing a finite-length schedule, *cf.* Figure 1(b). This schedule is assumed to be repeated periodically in time and we only constrain the average rate over the whole schedule. This constitutes a special case of the global problem (2), obtained by setting $\theta = 1$ and appropriately defining the constraints in (2d) such that an average-rate demand is fulfilled. More precisely, $D_u^{\max} = N$, $w_u = 1/U$, $\forall u \in \mathcal{U}$, each user has a single packet (*i.e.*, $S = U$, $\mathbf{d}^{u,(n)} \in \mathcal{R}_+$), and $\mathbf{s} \in \mathcal{R}_+^U$ contains the targeted average rates per frame. Neglecting delay costs is notably more insightful for investigating our algorithm as it makes the suboptimality of our convex approximation more visible.

We compare now the energy-expenditure per schedule as achieved by our algorithm to that using a constant rate over the same number of frames. On the physical layer we assume a VDSL upstream scenario with $U = 3$ users

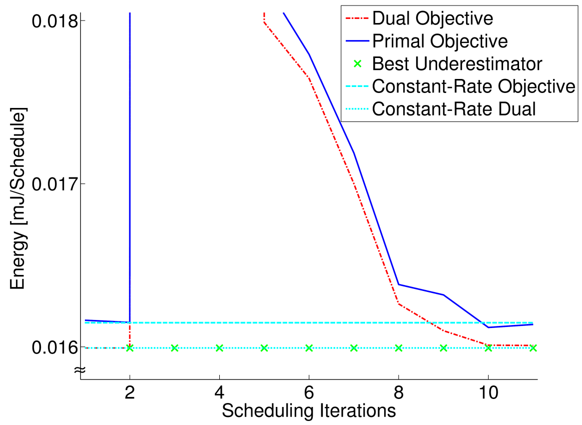

Fig. 3. Energy-Reduction by Periodic Scheduling for Interference Avoidance; Convergence of the BEEMS Algorithm in Dual Objective to the Best Found Underestimating Function Value And Comparison to a Constant Rate Solution.

located at 200 m/400 m/600 m distance from the deployment point, respectively. The simulation parameters were chosen according to the ETSI VDSL standard [16] (*i.e.*, an SNR-gap $\Gamma = 12.8\,\text{dB}$ and two upstream subbands as specified by band plan 997). The background noise comprises ETSI VDSL noise A added to a constant noise floor at $-140\,\text{dBm/Hz}$.

The simulation results for a schedule length of $N = 6$, a DMT system transmitting at 4000 frames/s and an average rate demand of 40 Mbps per user (*i.e.*, $s_u = N \cdot 10^4, \forall u \in \mathcal{U}$) are shown in Figure 3. We note that the scheduling cost is huge in iterations 3 and 4. This is due to infeasibility of the applied schedules which is punished by the modified spectrum management subproblems through a large cost, *cf.* Section III-A. As expected, the primal objective in our simulation closely follows the dual one. We also see that the best primal solution found by our algorithm outperforms the constant-rate solution. By (12) we find that the possible suboptimality of our algorithm can be upper-bounded by 0.8 %. Differently stated, our algorithm gave us a *certificate* that in this scenario *no time-sharing solution* could give an energy-improvement above 1 % compared to the energy-cost for constant-rate transmission.

We see that the schedule with constant rates over frames achieves a dual objective fairly close to the corresponding lower bound. We emphasize that this constant rate schedule can be constructed by solving a single subproblem (DSP$^{(n)}$) with average rate target. The solution of the linear problem (11) gives then already a "quick" valid upper bound on the gain by periodic rate scheduling. Thereby, we were able to obtain the above mentioned certificate by just solving a *single* spectrum management problem and a single linear program.

V. Conclusion

In this work we formulated an optimization problem for lazy scheduling and interference avoiding scheduling in interference-limited multi-user and multi-carrier digital subscriber line (DSL) systems. We proposed a cross-layer scheduling algorithm which is based on a problem approximation and a Benders decomposition approach. Our demonstration by means of an example indicates that the feasible energy-reduction through scheduling for interference-avoidance in DSL networks is negligible. This suggests that most of the potential power savings up to the link-layer are achieved by exploiting traffic variability. Furthermore, much simpler, single-user link-layer schedulers may be sufficient to harvest most of the possible performance gain. The latter however still necessitates more research on the design of low-complexity and real-time rate-adaptation algorithms for multi-carrier DSL systems.

A future target is to use the proposed algorithm for investigations on the possible energy-reduction by periodic schedules in *wireless* networks, where we expect higher benefits due to stronger interference. Another direction from this work is the extension to an infinite scheduling horizon through the derivation of (near-)optimal queue-state based rate policies.

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