Adaptive Subcarrier Allocation, Power Control, and Power Allocation for Multiuser FDD-DMT Systems

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Abstract— **This paper considers the problem of adaptive subcarrier allocation, power control, and power allocation for multiuser Frequency Division Duplex-Discrete Multitone (FDD-DMT) systems in a Gaussian interference channel. Assuming perfect knowledge of all channel and crosstalk transfer functions, we formulate the problem as an optimization problem to maximize jointly the sets of downstream and upstream bit rates for given user priorities. We show that the optimization problem belongs to the class of nonlinear mixed-integer optimization problems. We explain that for the multiuser FDD-DMT systems such problems can not be solved with existing algorithms. Instead, this paper introduces a new suboptimal normalized-rate iterative algorithm of low complexity.**

I. INTRODUCTION

During the last ten years various Digital Subscriber Line (DSL) technologies have established themself as viable methods for delivering broadband connections to the home. Currently, each DSL modem runs independently from all other modems and DSL modems are designed assuming a worst-case noise assumption regardless of the actual network environment. Thus, the data rates achieved are far below what is possible. An obvious extension for future DSL is to consider a cable bundle as a multiuser network. The analysis has shown that substantial performance improvement can be achieved by applying coordination and joint signal processing among the modems in a bundle [7]. Throughout this paper the term performance improvement is used to describe the increasing of bit rates that can be transmitted through the twisted pair channel.

On the other hand, to open up telecoms markets to new competition regulators in many countries have demanded unbundling. That is, the Competitive Local Exchange Carriers (CLECs) must be allowed to lease some telephone lines or some part of bandwidths from the Incumbent Local Exchange Carrier (ILEC). This will unfortunately render joint signal processing more or less impossible, because either the modems are allocated in different physical locations or the access to the physical-layer signals between the ILEC and different CLECs will not be allowed. However, performance can still be improved if DSL systems are analyzed as transmitting over a Gaussian interference channel. But for this channel the rate region is still an unsolved problem in information theory [1]. The rate region characterizes all possible data rate combinations among all users subject to the power constraints [9]. Anyway, there exists a distributed multiuser power control algorithm for DSL [9], which calculates the competitively optimal user power allocations. However, this algorithm does not consider the problem of the downstream and the upstream subcarrier allocation; instead it supposes that they are known and fixed. The convergence of the algorithm is, furthermore, only ensured if the user target bit rates are achievable, which has to be determined *a priori*.

In this paper we propose, in contrast to the distributed approach, a centralized "Normalized-Rate Iterative Algorithm", which determines an optimized downstream and upstream subcarrier allocation for the cable bundle and supported bit rates of all users for both transmission directions. Implicitly it performs power control and power allocation for all users in a competitively optimal way. Our algorithm does not need to know *a priori* the user target bit rates as we estimate the target bit rate adaptively in every iteration.

This paper proceeds as follows: First we analyze the DSL environment as a Gaussian interference channel and formulate the problem. Then in Section III we describe a new normalized-rate iterative algorithm. In Section IV we present the simulation results and analyze the performance of the algorithm and in Section V we draw our conclusions.

II. PROBLEM FORMULATION

We analyze the DSL environment as a multiuser environment with $2U$ transmitters and $2U$ receivers as in Fig. 1 (it is twice the number of the users U as we have duplex transmission). The modem located at the customer end is called Network Termination (NT), whereas the modem located in the other end, either in the Central Office (CO) or in the Cabinet (Cab) is called Line Termination (LT). We use the terms *downstream* when transmitting from an LT to an NT modem, and *upstream* for transmission in the opposite direction.

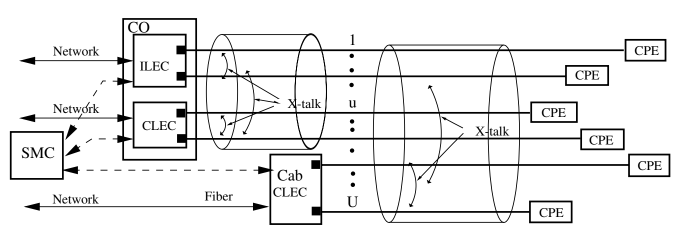

Fig. 1. A multiuser DSL environment with Costumer Premises Equipment (CPE) connected to a CO as well to a Cab.

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For a given transmission direction the receiver of the user u detects the data sent by the transmitter of the same user and treats the signals received from the other transmitters as noise. The background noise in DSL systems can be considered as additive white Gaussian noise and is typically smaller than the interference noise. This channel is known as a Gaussian interference channel.

We assume that synchronization is perfect and that full knowledge of all channel and crosstalk transfer functions is available to a central agent, which in the literature is called the Spectrum Management Center (SMC) [7]. The SMC can be located anywhere and can be managed by an independent entity. The ILEC and CLECs provide to the SMC center the channel and crosstalk transfer functions as well as the priorities of all users. From this data the subcarrier allocation, the power allocation, and the supported upstream and downstream bit rates for each user are calculated and sent back to all operators of the bundle.

Let us denote the downstream bit rate R_u^{DS} and the upstream bit rate R_u^{US} of user u on given DMT frame To maximize jointly the set of downstream bit rates $\{R_1^{DS}, R_2^{DS}, \ldots, R_U^{DS}\}$ and the set of upstream bit rates $\{R_1^{US}, R_2^{US}, \ldots, R_U^{US}\}$ for multiuser FDD-DMT systems the following parameters have to be determined for each user:

- Disjunctive sets of subcarriers w_u^{DS} and w_u^{US} used in the downstream and upstream directions; respectively;
- Used power in the downstream $P_{u,used}^{DS} \leq P_{u,max}^{DS}$ and upstream $P_{u,used}^{US} \leq P_{u,max}^{US}$ directions, where $P_{u,max}^{DS}$ and $P_{u,max}^{US}$ are the maximal power allowed to be used by user u in the downstream and upstream directions;
- Power distribution over used subcarriers (*i.e.*, Power Spectral Density (PSD) mask) in both transmission directions.

With the aim to offer different service classes to users we also introduce sets of downstream $\{\alpha_1^{DS}, \ldots, \alpha_U^{DS}\}$ and upstream $\{\alpha_1^{US}, \ldots, \alpha_U^{US}\}$ user priorities and a (a)symmetry factor a between the sum of downstream and sum of upstream bit rates of all users.

This can be formulated as an optimization problem:

$$\text{Maximize}\Big(\sum_{u=1}^{U} R_u^{DS} + \sum_{u=1}^{U} R_u^{US}\Big) \tag{1}$$

subject to: (2)

$$\sum_{n=1}^{N} w_{u,n}^{DS} P_{u,n}^{DS} \leq P_{u,max}^{DS}, \quad u = 1, \ldots, U \tag{3}$$

$$\sum_{n=1}^{N} w_{u,n}^{US} P_{u,n}^{US} \leq P_{u,max}^{US}, \quad u = 1, \ldots, U \tag{4}$$

$$\frac{R_1^{DS}}{\alpha_1^{DS}} = \frac{R_2^{DS}}{\alpha_2^{DS}} = \ldots = \frac{R_U^{DS}}{\alpha_U^{DS}}, \quad \sum_{u=1}^{U} \alpha_u^{DS} = 1 \tag{5}$$

$$\frac{R_1^{US}}{\alpha_1^{US}} = \frac{R_2^{US}}{\alpha_2^{US}} = \ldots = \frac{R_U^{US}}{\alpha_U^{US}}, \quad \sum_{u=1}^{U} \alpha_u^{US} = 1 \tag{6}$$

$$\sum_{u=1}^{U} R_u^{DS} = a \sum_{u=1}^{U} R_u^{US} \tag{7}$$

$$w_{u,n}^{DS} = 1 - w_{u,n}^{US}, \forall\ u, n; \text{ where } \begin{cases} u = 1, \ldots, U \\ n = 1, \ldots, N \end{cases} \tag{8}$$

$$w_{u,n}^{DS}, w_{u,n}^{US} \in \{0, 1\} \tag{9}$$

$$P_{u,n}^{DS}, P_{u,n}^{US} \in \{0, \quad ^{+}\}, \tag{10}$$

where N is the number of subcarriers, $P_{u,n}^{DS}$, $P_{u,n}^{US}$ and $w_{u,n}^{DS}$, $w_{u,n}^{US}$ are the transmit signal powers and the subcarrier usage indicators of user u in subcarrier n in the downstream and upstream directions, respectively. If there is a PSD mask constraint, the set $\{0, \quad ^{+}\}$ in Eq. (10) is replaced with the set $\{0, \ldots, P_{u,n,max}\}$. $P_{u,n,max}$ denotes the maximal power allowed to be used by user u in subcarrier n for a transmission direction.

The downstream bit rate of user u is calculated as:

$$R_u^{DS} = \sum_{n=1}^{N} w_{u,n}^{DS} R_{u,n}^{DS} \tag{11}$$

where $R_{u,n}^{DS}$ is the number of bits in the subcarrier n of user u in the downstream direction. For the upstream bit rate corresponding indexes in Eq. (11) are used.

To simplify the optimization problem in Eq. (1) we make the following assumptions:

A.1–Each subcarrier is assigned to either the upstream or the downstream direction and is used simultaneously by all users in that direction (if it used at all).
A.2–The subcarrier width Δf is fixed and equal for all users.
A.3–Any downstream subcarrier is orthogonal to any upstream subcarrier, implying that all modems work synchronously.

The A.1 assumption reduces the number of constraints dramatically, especially because the number of subcarriers used for VDSL systems is very large (up to 4092). With the assumptions A.1 - A.3 satisfied there is no self-near-end crosstalk (self-NEXT) noise and the noise in a subcarrier will depend only on the transmit power level of the other users on the same subcarrier. These assumptions simplify the constraints, as described below.

For a given subcarrier n, the user subcarrier variables $w_{u,n}^{DS}$ and $w_{u,n}^{US}$ in Eq. (3), (4), (8), (9) , and (11) are replaced with common subcarrier variable w_n^{DS} and w_n^{US}. Furthermore, Eq. (8) is rewritten as:

$$w_n^{DS} = 1 - w_n^{US}, \quad n = 1, \ldots, N. \tag{12}$$

Now, the number of bits in the subcarrier n of the user u in Eq.(11) (for a complex channel) is calculated as:

$$R_{u,n}^{DS} = log_2 \left(1 + \frac{|H_{uu,n}|^2 P_{u,n}^{DS}}{\Gamma\left(\sum\limits_{\substack{t=1 \\ t \neq u}}^{U} \left|H_{ut,n,FEXT}^{NT}\right|^2 P_{t,n}^{DS} + P_{n,BG} \right)} \right),$$

where Γ is the SNR gap, which shows how far a DSL modem is operating from the optimal Shannon capacity limit for a certain bit error rate. $H_{uu,n}$ is the channel transfer function of user u in subcarrier n. $H_{ut,n,FEXT}^{NT}$ denotes the far-end crosstalk channel transfer function from user t to user u in subcarrier n at NT side. $P_{n,BG}$ the background noise in subcarrier n.

The total bit rate expression in Eq. (1) as well as constraints in Eq. (5), (6), and (7) are neither convex nor concave with respect to user power allocations. Thus this optimization problem does not belong to the class of the optimization problems that are solvable with existing algorithms (see [3] for the solvable class of nonlinear mixed-integer optimization problems). Theoretically, one possible solution is to calculate the rate region by checking all possible user power combinations and all possible subcarrier allocations. However, the number of combinations is tremendously high and practically unfeasible. For instance, for a system with 2048 subcarriers only the number of subcarrier combinations is 2^{2048}.

III. Normalized-Rate Iterative Algorithm

Here we propose a normalized-rate iterative algorithm of low complexity that solves the problem formulated in Section II in a suboptimal way. It is considered suboptimal for two reasons: to make the algorithm tractable we have constraint it to search in a reduced space for subcarrier allocation; and it is based on Yu's iterative water-filling [8], which finds the competitively optimal power allocation solutions, which are, for the Gaussian interference channel, not globally optimal [9].

Our algorithm has two main phases: *initialization* and *iteration*. The iteration phase is further divided into two stages: *an outer stage* and *an inner stage*.

The outer stage searches for the subcarrier allocation in the downstream or upstream direction, *c.f.*, Eq.(12). In principle, each subcarrier should be allocated in the direction of higher average channel-gain-to-noise ratio. However, the noise in subcarrier n is unknown *a priori* because it depends on the power allocation of all other users in the same subcarrier.

In our algorithm the subcarrier allocation starts with the initial values, where the bandwidth from N subcarriers is partitioned into K subbands with an equal number of subcarriers per subband. The subbands are allocated in the downstream and upstream directions in successive order. A binary search within the subbands for the downstream or upstream subcarrier allocation is performed (simultaneously over all users as we have FDD with the same subcarrier allocation for all users) until Eq. (7) is satisfied to a desired accuracy. This can be done by simultaneously moving either all downstream right subband edges in Fig. 2 denoted by bullets (an example with four subbands) or all downstream left subband edges.

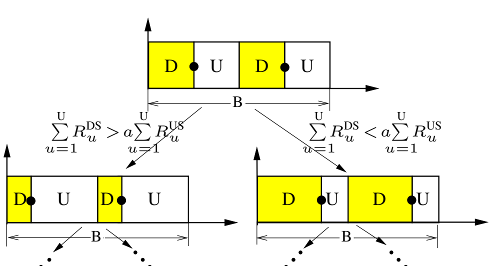

Fig. 2. Searching for subcarrier allocation.

The inner stage calculates the supported bit rate of each user as well as implicitly performs power control and power allocation for all users. The inner stage is based on Yu's [8] iterative water-filling algorithm with the target bit rate estimated in every iteration. To do this we have defined a normalized supported bit rate, which is calculated as the supported bit rate of a given user u divided by the user priority α_u. The target bit rate is the bit rate that is aimed to achieve in the iteration i for a given maximal power allowed to be used. We will use the linear least squares estimator [4] to calculate the target bit rate because we cannot make *a priori* any probabilistic assumption about the downstream and upstream supported bit rates of any user. Thus, the target bit rate is calculated as the mean value of the normalized supported bit rates in the last m iterations multiplied by user priority α_u. The algorithm works well using $m = U$. However, performance improvement sometimes is achieved when m is increased. To achieve the best performance and the fastest convergence m should be updated adaptively. In the following of this paper we assume that only fixed m value is used.

Due to the estimation of the target bit rate in every iteration, the user order over which we iterate becomes important for the algorithm's performance. It is quickly realized that the users should be arranged first in decreasing priority order and within the same priority group they should be arranged in decreasing line-attenuation order. We perform this ordering independently for both transmission directions.

The water-filling algorithm used in the inner stage is a modified version of the fixed-margin [7] water-filling algorithm. In our case we do not know *a priori* if the target bit rate that can be supported for a given maximal power allowed to be used. Therefore we have modified the fixed-margin water-filling algorithm as follows: if the target bit rate can be supported, then only the power needed to support that given bit rate is used; otherwise the maximum allowed power is used and the supported bit rate is calculated.

The algorithm for the general case is presented below:

<u>Initialization phase:</u>

I) For a given number of subcarriers N, the number of subbands is initialized to $K = 2^b$, where b can take one of the following values: $1, 2, \ldots, log_2 N$. The sets of subcarriers for the upstream w^{US} and downstream w^{DS} directions are initialized from the initial allocation of the K subbands.

II) Set the downstream PSD mask P_u^{DS} and upstream PSD mask P_u^{US} of all users to zero.

III) Initialize the sets of user priorities for the downstream $\{\alpha_1^{DS}, \ldots, \alpha_U^{DS}\}$ and upstream $\{\alpha_1^{US}, \ldots, \alpha_U^{US}\}$ directions, together with the asymmetry parameter a.

IV) The users are first arranged in decreasing priority order. Then users within the same priority group are arranged in decreasing line-attenuation order. The ordering is performed independently for both transmission directions.

V) Set the initial target bit rates $\tilde{R}_{Target}^{DS}$ and $\tilde{R}_{Target}^{US}$ to infinity.

VI) Set the parameters m^{DS} and m^{US} (which specify the last m normalized supported bit rates used to calculate the target bit rate for a given transmission direction) to user-selected values.

Iteration phase:

1. Repeat (outer stage).
2. Set the downstream iteration counter to $i^{DS} = 1$.
3. Repeat (downstream inner stage).
4. Repeat for all users u (in the same order as in IV).
5. Calculate the noise for the set of subcarriers w^{DS} at the NT side. For a given user u in subcarrier n it is calculated as:

$$N_{u,n}^{NT} = \sum_{\substack{t=1 \\ t \neq u}}^{U} \left| H_{tu,n,FEXT}^{NT} \right|^2 P_{t,n}^{DS} + P_{n,BG}. \qquad (13)$$

6. The target bit rate is updated and it is calculated as (for $i^{DS} = 1, R_{Target}^{DS}$ is equal to the initial value in V):

$$\tilde{R}_{Target}^{DS} = \frac{\alpha_u^{DS}}{r} \sum_{k=i^{DS}-r}^{i^{DS}-1} \overline{R}_k^{DS}, \qquad (14)$$

where $r = \begin{cases} i^{DS} - 1 & \text{if} \quad i^{DS} \leq m^{DS} \\ m^{DS} & \text{if} \quad i^{DS} > m^{DS}. \end{cases}$

7. Apply modified fixed-margin water-filling to user u for the target bit rate calculated in Step 6 and defined $P_{u,max}^{DS}$. As a result the supported bit rate R_u^{DS} and the corresponding PSD mask $\hat{P}_u^{DS}$ of the user u are found.
8. Update the downstream PSD mask of user u: $P_u^{DS} = \hat{P}_u^{DS}$
9. The downstream normalized supported bit rate in iteration i^{DS} is calculated as:

$$\overline{R}_{i^{DS}}^{DS} = \frac{R_u^{DS}}{\alpha_u^{DS}}.$$

10. Increase the downstream iteration counter $i^{DS} = i^{DS} + 1$
11. Go to (4). Repeat for all users.
12. Go to (3). Repeat until the desired accuracy on the user supported bit rates is reached.
13. Repeat Steps 2 to 12 for the upstream direction.
14. Depending on the user downstream and upstream supported bit rates, and the asymmetry constraint a the subband edges are moved (*c.f.*, Fig. 2). Then the set of subcarriers for the downstream w^{DS} and for the upstream w^{US} direction are calculated for the next outer stage iteration.
15. Go to (1). Repeat until the desired (a)symmetrical accuracy is achieved in Eq. (7) or the maximal number of combinations have been examined.

IV. Simulation Results

In this section we present simulation results and analyze the impact of the parameters K, m^{DS}, and m^{US} on the performance of the Normalized-Rate Iterative Algorithm.

For our simulations we use the network scenario shown in Fig. 3. Most simulation parameters are based on the ETSI VDSL specification [2]. Thus the cable model used is a 0.5 mm copper cable TP150; the maximal power allowed to be used by each user in both transmission directions is $P_{u,max}^{DS} = P_{u,max}^{US} = 11.5$ dBm; $FEXT$ coupling $K_{FEXT} = -45$ dB at 1 MHz; the background noise is set to flat level of $P_{BG} = -140$ dBm/Hz; the SNR gap is set to $\Gamma = 12.3\,dB$; the subcarrier width is set to $\Delta f = 4.3125$ kHz; the number of subcarriers is $N = 2048$; the number of frames per second is 4000; the cyclic extension length is set to 320 samples; there is no constraint in the maximal number of bits per subcarrier and any real number of bits can be transmitted; there is no PSD mask constraint. For all simulations we assume that all users have the same priority on both transmission directions and the asymmetry constraint is set to $a = 1$. To search for the subcarrier allocation we use the binary search method shown in Fig. 2.

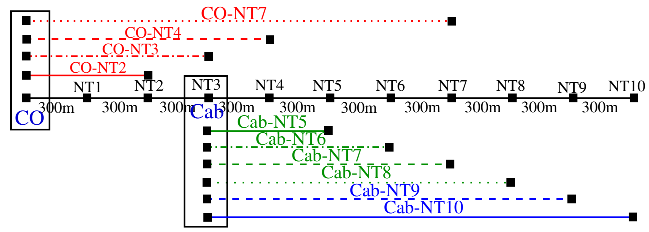

Fig. 3. Simulation scenario.

Example 1: Here we will consider the impact of the number of subbands on the algorithm's performance. For all simulations in this example m^{DS} and m^{US} are set equal to 100 (ten times of number of modems in the simulation scenario). This increases the number of iterations but assures the maximal achievable user bit rates. We perform simulations for $K = 2, 4, 8, 16$, and 32 subbands.

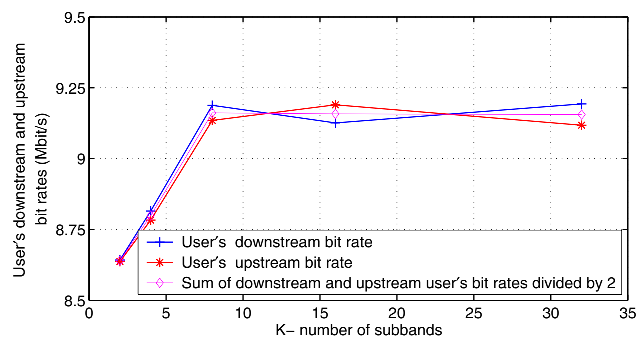

Fig. 4. User's downstream and upstream bit rates for various numbers of subbands K.

In Fig. 4 we see only a minor increase in the sum of downstream and upstream bit rates when K is increased above 8. Therefore the value of $K = 8$ or $K = 16$ for the network scenarios we have simulated seems a good choice. We also see that, as a result of the simultaneous movement of "subband edges", the accuracy in supported downstream and upstream users bit rate decreases with more subbands K. If needed, it is possible to extend the algorithm to increase the accuracy of user's downstream and upstream bit rates by fine-tuning of a single edge after the multi-edge movement has converged. However, the number of required iterations then increases and improvements from practical point of view are minor.

The subcarrier allocation for $K = 8$ subbands was found to be: $w^{DS} = \{2\text{–}149, 513\text{–}661, 1025\text{–}1173, 1537\text{–}1685\}$ for the downstream direction and $w^{US} = \{150\text{–}512, 662\text{–}1024, 1174 \text{–}1536, 1686\text{–}2047\}$ for the upstream direction.

Example 2: In this example we consider the impact of m^{DS} and m^{US} on the algorithm performance and the algorithm convergence behavior (m^{DS}, m^{US} is the number of the last normalized supported bite rates to use when calculating the target bit rate). We fix the number of subbands to $K = 8$ and use the resulting subcarrier allocation for the downstream and upstream direction found in *Example 1*. As shown in Fig. 5

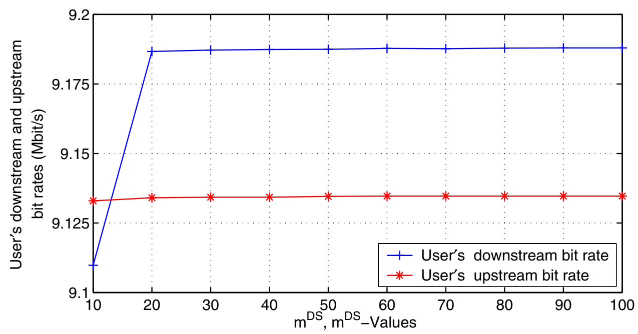

Fig. 5. User's downstream and upstream bit rates depending on the number of the last m^{DS} and m^{US} normalized supported bit rates used to calculate the target bit rate.

there is only a small improvement in the user's supported bit rate if m^{DS} and m^{US} are increased (this is due to the user ordering as specified in initialization phase).

The algorithm convergence behavior for upstream direction is shown in Fig. 6 for $m^{US} = 10$. It can be seen that the algorithm converges exponentially.

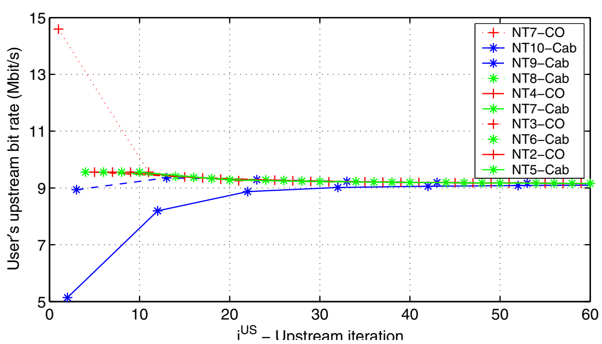

Fig. 6. Upstream convergence behavior of normalized-rate iterative algorithm.

Example 3: As mentioned earlier, there are no algorithms that calculate the optimal power allocation and subcarrier allocation for multiuser FDD-DMT systems. Therefore we compare the performance of our proposed algorithm with an exhaustive search algorithm for subcarrier allocation. To make the exhaustive search tractable a certain number of subcarriers are grouped into a subband. Simulations are carried out for $K = 16$ subbands with an equal number of subcarriers per subband. With our proposed algorithm simulations are carried out for $K = 8, 16$, and 32 subbands. Simulation results are presented in Fig. 7 and they show that the performance of our proposed algorithm is the same to the performance

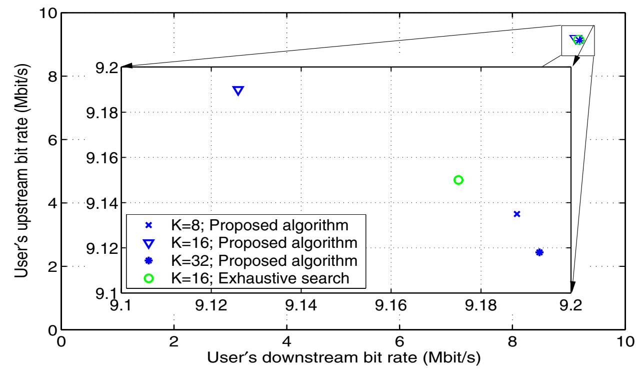

Fig. 7. Comparison of our proposed algorithm with exhaustive search algorithm for the optimal subband allocation when $K = 8$ subbands.

of an exhaustive search algorithm for the optimal subband allocation.

V. Conclusion

In this paper we have considered the problem of adaptive subcarrier allocation, power control, and power allocation for multiuser Frequency Division Duplex-Discrete Multitone (FDD- DMT) systems in a Gaussian interference channel. We show that such problems cannot be solved with existing algorithms. Therefore, we have developed a novel *normalized-rate iterative algorithm* of low complexity that (suboptimally) solves this problem. By simulation we have shown that the algorithm converges rapidly and achieves the same bit rates as an exhaustive search algorithm for subcarrier (subband) allocation.

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