Spectrum Balancing for DSL with Restrictions on Maximum Transmit PSD

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***Abstract*—The importance of a power spectral density (PSD) mask restriction is often overlooked when optimizing the spectrum usage for multiuser digital subscriber lines (DSL) systems. However, by developing the optimization strategies based only on the PSD constraints (masks) we can tremendously reduce the computation complexity compared to the methods only based on the total power restriction. In this paper we introduce a mask-based spectrum balancing (MSB) algorithm and demonstrate the near optimum performance of this optimization approach. Furthermore, we show that besides standards compliance, PSD restriction is also needed to ensure the convergence of iterative spectrum balancing methods, which use dual decomposition optimization.**

I. Introduction

In recent years a number of spectrum balancing methods have been introduced for digital subscriber line (DSL) systems in order to optimize the spectrum utilization for multiuser network scenarios. The problem is to design the transmit spectra for U DSL systems (users) sharing a cable bundle and thus interfering with each other. This is typically described as an optimization problem where the aim is to maximize the sum of weighted bitrates. Such an optimization is usually constraint by some limits on the used power. Two approaches exist: one is to restrict the total power and the other is to set a PSD mask where the power is restricted over frequency. These two approaches can also be combined.

Since the introduction of DSL transmission techniques, 20 years ago, standardization bodies have set both kinds of limits on the used power. The total power limit is mostly aimed at reducing power consumption and lessening the demands on the analog front-end. The PSD limits are mostly set for radio frequency interference egress and to reduce the disturbance of other (legacy) DSL systems. In all standards up till now, including the latest very high-speed DSL standard known as VDSL2, the total power under the PSD mask aims to match the total power allowed. Therefore it seems reasonable to base the spectrum balancing optimization solely on the PSD mask instead on total power. Exploring this concept, we have developed a new spectrum balancing algorithms that we call mask-based spectrum balancing (MSB). Furthermore, our motivation was driven by instability problems we have encounter when implementing the iterative spectrum balancing (ISB) [1], [2] which only uses a total power constraint.

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When evaluating spectrum balancing algorithms the network scenario (the number of users and cable topology) might skew the results in various directions. For example, if only two users are selected (might be good for reasons like computational convenience or being able to visualize the rate regions) methods that can emphasizes frequency division multiplexing, like optimal spectrum balancing (OSB) [3] and ISB, shines. If we instead have a scenario with many users they will all fight for the used frequencies and no one gets a frequency by itself, and the simple methods like iterative water-filling [4] or normalized-rate iterative algorithm [5] will do almost as well as the optimal ones.

In this paper we will still use a two user scenario. We do this mostly for rate region visualization reasons. This will be to the disadvantage of MSB, as discussed above, and to our experience MSB becomes even closer to the optimum when more users are included into the simulation scenario.

Another problem when doing these kind of evaluations is that the largest benefit with OSB is for scenarios with long distance between nodes and when total power is utilized, that is, for long lines. Thus, few investigations has been made for realistic scenarios for cabinet deployed VDSL2 (max distance of 500-800 m). However, when such scenarios are tested, OSB and ISB behaves unexpectedly and shows convergence problems. To highlight these problems and because we think it is more realistic scenario, we will also simulate a VDSL2 cabinet deployment scenario with relatively short loops.

The paper is structured as follows. In the following section we show how to calculate bitrates in multiuser DSL environments and give a short introduction to the dual decomposition based optimization for DSL. Then in Section III we describe the need for PSD restriction and consequences for not including it into optimization. In Section IV we describe our mask-based spectrum balancing (MSB) algorithm and following that we present some simulation results where we compare MSB with ISB. In the end we give some concluding remarks.

II. Preliminaries

In this paper we will only analyze DSL systems that use a frequency division duplex (FDD) transmission scheme. For such systems, based on Shannon's capacity formula the number of total bits that can be transmitted in a discrete multi-tone (DMT) symbol (at a certain bit-error rate) for a particular user u is determined as

$$R_u = \sum_{n \in I} \log\left(1 + \frac{\mathcal{H}_{uu}^n \mathcal{P}_u^n}{\Gamma \mathcal{N}_u^n}\right), \quad \text{with} \tag{1}$$

$$\mathcal{N}_u^n = \sum_{\substack{v=1 \\ v \neq u}}^{U} \mathcal{H}_{uv}^n \mathcal{P}_v^n + \mathcal{P}_{u,V}^n, \tag{2}$$

where I denotes the set of subcarriers used in a particular transmission direction and it comprises N subcarrier; Γ is the signal-to-noise ratio gap; $\mathcal{N}_u^n$, $\mathcal{P}_u^n$, $\mathcal{P}_{u,V}^n$ denote the PSD of user u in subcarrier n of noise, transmit signal, and the sum of background and alien noises, respectively; $\mathcal{H}_{uv}^n$ denotes the squared magnitude of channel transfer function from user v to user u, *i.e.*, it represents either the direct channel (with $v = u$), or far end crosstalk (FEXT) coupling.

As shown in [3], the optimization goal for DSL can be formulated as an optimization problem where the objective is to maximize the sum of weighted bitrates:

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}} \quad \sum_{u=1}^{U} w_u R_u, \tag{3a}$$

$$\text{subject to:} \quad \sum_{n \in I} \mathcal{P}_u^n \leq T_u^{\max}, \quad \forall\ u, \tag{3b}$$

$$\mathcal{P}_u^n \geq 0, \forall\ u, \forall n \in I \tag{3c}$$

where w_u denotes the weighting value (or short: 'weight') assigned to user u and $T_u^{\max}$ denotes the total power constraint for user u. Without loss of generality, the weights can be selected such that $\sum_{u=1}^{U} w_u = 1$. We increase the bitrate of user u compared to the bitrates of the other users by increasing its w_u. The total power constraint $T_u^{\max}$ is usually selected to be the same for all users.

To solve this constrained optimization problem, the total power constraint from (3b) is incorporated into the cost function (3a) by defining the Lagrangian function [6]:

$$L = \sum_{u=1}^{U} w_u R_u + \sum_{u=1}^{U} \lambda_u \left(T_u^{\max} - \sum_{n \in I} \mathcal{P}_u^n\right), \tag{4}$$

where λ_u denotes the Lagrangian multiplier of user u. Based on the definition of Lagrangian, the optimization problem (3) can be written as

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}} \quad L(w_u, \lambda_u, \mathcal{P}_u^n), \tag{5a}$$

$$\text{subject to:} \quad \mathcal{P}_u^n \geq 0, \forall\ u, \forall n \in I \tag{5b}$$

$$\lambda_u \geq 0, \forall\ u. \tag{5c}$$

By collecting the terms that belong to the same subcarrier (4) can be rewritten as

$$L = \sum_{n \in I} L^n + \sum_{u=1}^{U} \lambda_u T_u^{\max}, \tag{6}$$

where L^n is the Lagrangian on subcarrier n and is given by

$$L^n = \sum_{u=1}^{U} w_u R_u^n - \sum_{u=1}^{U} \lambda_u \mathcal{P}_u^n. \tag{7}$$

Such an approach in optimization is known as dual decomposition, where the optimization is divided into N per-subcarrier optimization subproblems that are only related through the weighs w_u and Lagrangian multipliers λ_u. This in fact leads to an optimization which has a complexity that scales linearly with the number of subcarriers. However, note that solving directly optimization problem (5), as has been done in [3], still has an complexity that increases exponential with the number of users (lines).

From the theory of the Lagrangian functions it is known that at the convergence point, Lagrangian multipliers will be selected such that either the total power of user u satisfies $\sum_{n \in I} \mathcal{P}_u^n = T_u^{\max}$ or $\lambda_u = 0$. We achieve different bitrate combination among the users by changing weights assigned to them. By trying out all weights combination among the users, we gain the boundary of the bitrate region. The reason for this is that by dual decomposition optimization only bitrates that lies on the rate region boundary can be found, since only these bitrates maximize the sum of weighted bitrates.

III. The Need for Constraining the PSD

To solve the optimization problem (5) a number of algorithms have been proposed: optimum spectrum balancing (OSB) [3], iterative spectrum balancing (ISB) [1], [2], autonomous spectrum balancing (ASB) [7], and successive convex approximation for low-complexity (SCALE) [8]. For all these algorithms it is claimed that there is no need for the maximum transmit PSD mask constraint, since total power constraint, $T_u^{\max}$, is thought to be sufficient to ensure the spectral compatibility among the DSL systems included into optimization. However, in this section we will show that this assumption might not be valid under certain circumstances.

Due to low computational complexity and performance almost identical to the OSB, iterative-based algorithms have received a special attention for spectrum balancing. Therefore, we will constraint ourself to ISB which works as follows: it iterates many times over all users and in each iteration it maximizes (7) for one user while keeping the power allocation of all other users fixed. At the end of each iteration the weight and Lagrangian multiplier of the user being optimized are updated, cf. Algorithm 1.

In the following discussions, we constraint ourself to two-user case for easy explanations, but all the claims made here are also valid for scenarios with multiple users. Now looking at the i-th iteration, for a two-user case, the optimization problem for the first user on a particular subcarrier n is

$$\underset{\mathcal{P}_1^n}{\text{maximize}} \quad \sum_{u=1}^{2} w_u R_u^n - \sum_{u=1}^{2} \lambda_u \mathcal{P}_u^n, \tag{8a}$$

$$\text{subject to:} \quad \mathcal{P}_1^n \geq 0 \tag{8b}$$

As shown in [1], [2], the cost function (8a) is neither concave nor convex with respect to power allocation of the first user $\mathcal{P}_1^n$. Therefore, the exhaustive search is used to find $\mathcal{P}_1^n$, which maximizes (8a). Let us analyze the case when $\mathcal{P}_2^n = 0$. This occurs whenever the SNR of the second user on the subcarrier n is low either due to high noise level or high channel attenuation. For this case, the optimization problem (8) becomes

$$\underset{\mathcal{P}_1^n}{\text{maximize}} \;\; w_1 \log\left(1+\frac{\mathcal{H}_{11}^n \mathcal{P}_1^n}{\Gamma \mathcal{P}_{1,V}^n}\right) - \lambda_1 \mathcal{P}_1^n \tag{9a}$$

$$\text{subject to: } \mathcal{P}_1^n \geq 0. \tag{9b}$$

The second derivative of (9a) can be calculated in a closed form and it is equal to

$$-\frac{w_1 \left(\mathcal{H}_{11}^n\right)^2}{\left(\Gamma \mathcal{P}_{1,V}^n + \mathcal{H}_{11}^n \mathcal{P}_1^n\right)^2}. \tag{10}$$

Since (10) is smaller than zero for $0 \leq \mathcal{P}_1^n < \infty$, the cost function in (9a) is a concave function with respect to $\mathcal{P}_1^n$. For this case, there is still no need for a maximum transmit PSD mask constraint to ensure the convergence of ISB. However, Proposition 1 below shows that in order to ensure convergence of ISB in any network scenario we need to set-up a PSD mask constraint.

Algorithm 1 Iterative Spectrum balancing as in [1] (*Notation is adopted for this paper*)

Preset values: $R_u^{\text{target}}, T_u^{\max}, \lambda_u, w_u, \;\; \forall u$
repeat
  for $u = 1$ to U **do**
    repeat
      For each n: fix $P_j^n, \forall j \neq u$, then
      $P_u^n = \arg\max_{\mathcal{P}_u^n} L^n$ for $n \in I$ as in (6)
      {Solve by 1-D exhaustive search }
      Update: $w_u = \left[w_u + \epsilon\left(R_u^{\text{target}} - \sum_{n\in I} R_u^n\right)\right]^+$
      Update: $\lambda_u = \left[\lambda_u + \epsilon\left(\sum_{n\in I} P_u^n - T_u^{\max}\right)\right]^+$
      {$[]^+$: constraint to non-negative numbers}
    until convergence
  end for
until the PSDs of all users have reach a desired accuracy

Proposition 1: ISB algorithm converges in any network scenario only if we set a maximum transmit PSD mask constraint.

Proof: We will show that ISB does not converge when both $\lambda_1 = 0$ and $\mathcal{P}_2^n = 0$ (after the proof we describe when such a situation arises). Note that for optimization problem (8) the same also holds when $\lambda_2 = 0$ and $\mathcal{P}_1^n = 0$.

With the made assumptions, the optimization problem (9) becomes

$$\underset{\mathcal{P}_1^n}{\text{maximize}} \;\; w_1 \log\left(1+\frac{\mathcal{H}_{11}^n \mathcal{P}_1^n}{\Gamma \mathcal{P}_{1,V}^n}\right) \tag{11a}$$

$$\text{subject to: } \mathcal{P}_1^n \geq 0. \tag{11b}$$

The cost function (11a) is monotonically increasing on $\mathcal{P}_1^n$. Therefore, the optimization problem (11) achieves a maximum at $\mathcal{P}_1^n = \infty$, and the search for it will not converge. Thus, to ensure the convergence of ISB in any network scenario, we have to set a PSD mask constraint, which also concludes the proof. ■

We describe now for which cases both λ_1 and $\mathcal{P}_2^n$ get zero, based on ISB's outer iterations (outer repeat loop) as given in the pseudo-code of Algorithm 1. Assume that in the $(i-3)$-th iteration, second user performs power allocation and it utilizes all subcarriers in set I. Furthermore, in $(i-2)$-th iteration, first user does not utilize the total power to achieve a particular target bitrate, R_1^{target}; thus, $\lambda_1 = 0$. In $(i-1)$-th iteration, the second user search for a new power allocation, and it does not use at least one subcarrier. In (i)-th iteration, we start to calculate the optimal power allocation for the first user with $\lambda_1 = 0$ and there is at least an n for the second user which has $P_2^n = 0$.

IV. Optimization under a maximum PSD mask constraint

As we have shown in Section III, the convergence of ISB is only ensured if we set a maximum transmit PSD mask constraint in addition to the total power constraint. In this section we show that the optimization problem for DSL systems is tremendously simplified if we select the total power constraint to be equal to the power within the transmit PSD mask constraint. Under this assumption, the spectrum balancing optimization problem can be formulated as

$$\underset{\mathcal{P}_u^n;\forall u,n}{\text{maximize}} \;\; \sum_{u=1}^{U} w_u R_u, \tag{12a}$$

$$\text{subject to: } 0 \leq \mathcal{P}_u^n \leq \mathcal{P}_u^{n,\max}, \forall\, u, \forall n \in I, \tag{12b}$$

where $\mathcal{P}_u^{n,\max}$ denotes the maximum transmit PSD mask constraint for user u in subcarrier n. Following this approach by setting a unique transmit PSD mask constraint for each user (line), we can include modems's implementation constraints into the optimization process. Furthermore, setting a unique transmit PSD mask constraint for each user does not increase the computational complexity. However, using the same maximum transmit PSD constraint for all users is only a special case of optimization problem (12).

Maximizing the bitrate in each subcarrier independently also maximizes the sum of bitrates over all subcarrier. Therefore, the optimization problem (12) can be split into N per-subcarrier optimization subproblems by expressing the cost function as

$$\sum_{u=1}^{U} w_u R_u = \sum_{n \in I} \sum_{u=1}^{U} w_u R_u^n. \tag{13}$$

Different methods have been proposed to search for the appropriate weighting values w_u. Under the assumption that the target bitrates of all users are known in advance (before running the algorithm), a sub-gradient method is proposed in [1]. Alternatively the algorithm in [5] uses bitrate relations in order to search for the weights without any *a priori* knowledge on target bitrates. As the search for appropriate weights is not the focus of this paper we will in the following not search for the weights but instead we vary them between 0 and 1 to picture the complete rate region.

The pseudo-code of our proposed iterative scheme, which we call mask-based spectrum balancing (MSB), is listed as Algorithm 2. The MSB works as follows: For given weights and PSD mask constraints, it iterates many times over all users and in each iteration it searches for the transmit PSD of a particular user that maximizes the sum of weighted rates.

Algorithm 2 Mask-based spectrum balancing (MSB)

Preset values: $w_u, \quad \forall u$
$P_u^{n,\max}, \quad \forall u, \forall n \in I$ {mask constraints}
repeat
  for $u = 1$ to U **do**
    Calculate Noise $\mathcal{N}_u^n$ for $n \in I$ as in (2)
    $P_u^n = \arg\max_{\mathcal{P}_u^n} \sum_{u=1}^{U} w_u R_u^n$ for $n \in I$ as in (6)
    {Solve by 1-D exhaustive search under constraint (12b)}
  end for
until the PSDs of all users have reach a desired accuracy

MSB is not the first algorithm to state a PSD mask constraint. For example, the user unique power back-off (UUPBO) [9] solves a similar optimization problem as ISB. However, it sets an constraint on maximum transmit PSDs in addition to the total power constraint. This ensures the convergence of UUPBO in any network scenario.

V. Simulation Results and Discussions

In order to evaluate the performance of our proposed algorithm, mask-based spectrum balancing (MSB) algorithm, simulations have been used. Simulation parameters are taken according to ETSI VDSL standard [10]. Thus, we use $\Gamma = 12.8\,$dB as the SNR gap, and the band plan 997, which uses two upstream bands. Moreover, to take into account the alien noise, in addition to the background noise at $-140\,$dBm/Hz, we have also added the ETSI VDSL Noise A, which is also specified in [10].

We compare the performance of the MSB with the iterative spectrum balancing (ISB) for the upstream transmission direction. For ISB we have set a PSD mask constraint to $0\,$dBm/Hz, which is only used to ensure the convergence of ISB in the light of discussion in Section III. For MSB we have set a PSD mask constraint to $-59\,$dBm/Hz for all simulations. To have a fair comparison between MSB and ISB, the total power constraint by ISB was set to be equal to power within the PSD mask constraint in MSB. All simulation are performed for the network scenario shown in Fig. 1 with only two users. For such two-user case the bitrate region is two dimensional and is easy to draw conclusions from. Furthermore, to cover a broad range of network environments, simulations are done for the distance x between the modems equal to: 200 m, 400 m, and 600 m.

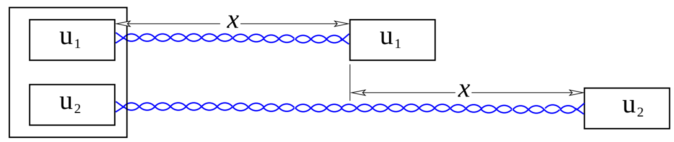

Fig. 1. Simulation scenario with two users. CO and x denote the central office and the distance between the modems, respectively.

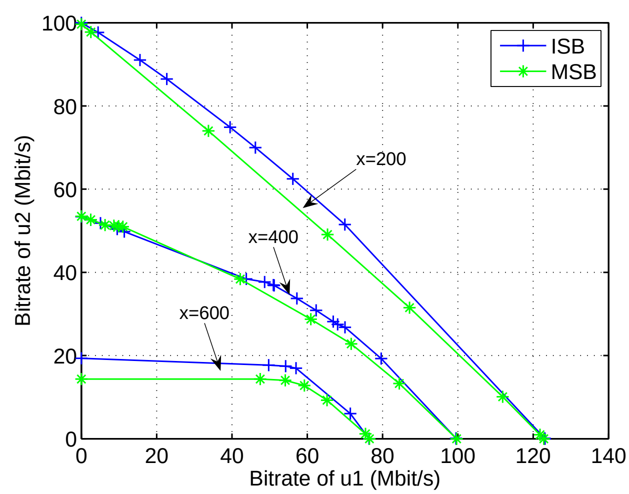

Fig. 2. Comparison of the rate regions between the MBS and ISB for upstream transmission direction. Simulations are performed for the network scenario with two users as in Fig. 1 and the distance x between the modems equal to: 200 m, 400 m, and 600 m.

From the simulation results presented in Fig. 2 it is obvious that ISB outperform MSB for these network scenarios. The results are not surprising, since ISB uses more degrees of freedom and it utilizes them when searching for optimal transmit PSDs. In Table I a few, from Fig. 2, selected pairs of bitrates are shown.

For the network scenario with $x = 600\,$m we see in Fig. 2 that ISB visibly outperforms MSB when the operation point is selected on the left side of rate region. For the pairs of bitrates shown in Table I, ISB outperforms MSB by 6.6% . For this set of bitrates the transmit PSDs are shown in Fig. 3. The transmit PSDs of ISB are always above the transmit PSDs of MSB, which is also the reason why ISB outperforms MSB.

TABLE I
COMPARISON OF THE MSB WITH THE OSB FOR SOME PARTICULAR PAIRS OF BITRATES.

Algorithm	Scenario (x in m)	User u_1 (Mbit/s)	User u_2 (Mbit/s)	Loss (%)
ISB	600	62.2	14.6	–
MSB	600	59.2	12.8	6.6
ISB	400	82.0	17.0	–
MSB	400	80.8	16.0	2.2
ISB	200	107.5	15.0	–
MSB	200	107.0	14.3	3.0

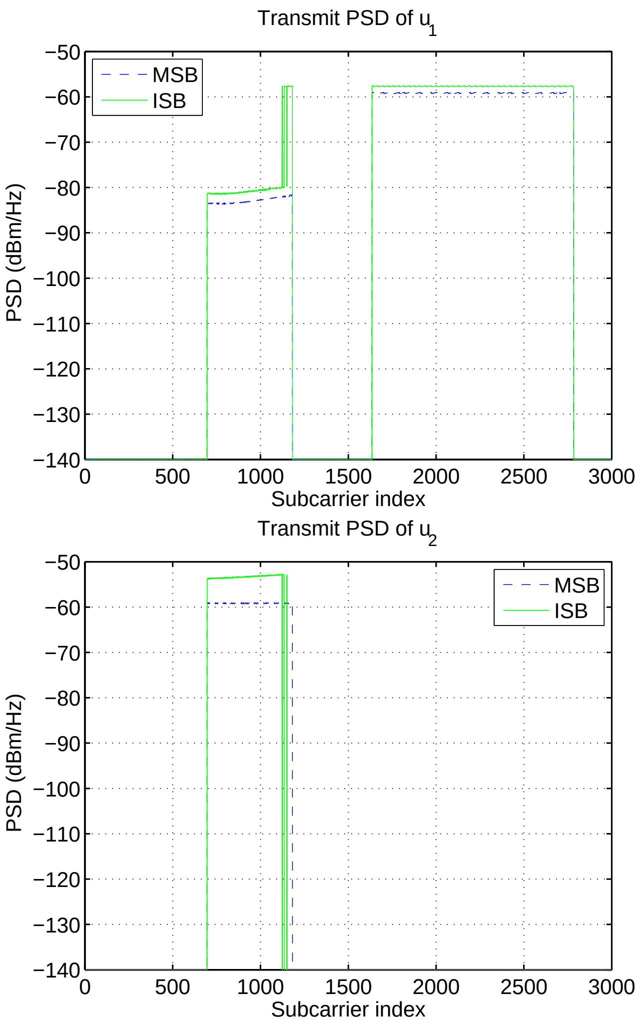

Fig. 3. Upstream transmit PSDs of MSB and ISB for users' bitrates given in Table I for $x = 600$ m.

On the other hand, for the network scenario with $x = 400$ m and $x = 200$ m, ISB and MSB show similar performance. For these pairs of bitrates shown in Table I, ISB only outperforms MSB by 2 to 3 percent. Furthermore, for $x = 400$ m the boundary of the rate regions for ISB and MSB as plotted are not convex. The reason for this effect is that the double precision arithmetic, which is also used during simulations is not always sufficient to find every operation point on the boundary of the rate region. It is worth mentioning that this holds also for other spectrum balancing algorithms based on the dual decomposition optimization approach.

Comparing the pseudo-code of MSB and ISB, it becomes clear that ISB requires much higher computational complexity. To indicate the difference in complexity, the simulation time to get a pair of bitrates is 3 seconds for MSB while ISB (for fixed weights) requires 114 seconds.

VI. CONCLUSIONS

This paper introduced a new spectrum balancing algorithm for multi-user DSL systems: the mask-based spectrum balancing (MSB) algorithm. The optimization by MSB is only based on PSD masks constraint instead on the total power constraint as used by other spectrum balancing algorithms. We have shown that an optimization approach based on PSD masks instead on the total power makes sense both from a computational complexity and a stability point of view. The stability issue of commonly used spectrum balancing methods is further analyzed and we have proven that stability issues exist. Through simulations we have additionally shown that the inevitable loss by MSB compared to the optimal schemes (due to less degrees of freedom) is kept very low and is only a few percent for scenarios with short loops (suitable for cabinet deployed VDSL2). This small loss should be compared to the complexity reduction of two magnitudes for MSB compared to ISB.

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