The Constrained Normalized-Rate Iterative Algorithm

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***Abstract*— Dynamic spectrum management (DSM) aims to increase the utilization of cable capacity by adapting the spectra of digital subscriber line (DSL) systems to the real network environment. In this paper we present a new DSM algorithm: the constrained normalized-rate iterative algorithm (C-NRIA). The C-NRIA extends the existing NRIA by ensuring predefined fixed bitrates to some of the users in the cable bundle while offering bitrates to the remaining users on a best effort basis. This reflects many business scenarios where a number of users must be guaranteed a specific service. We show that this type of the optimization problem can be solved efficiently with the C-NRIA by introducing only *one* balancing parameter that splits the cable capacity among the two user groups.**

I. Introduction

The normalized-rate iterative algorithm (NRIA) [1], [2] is a centralized dynamic spectrum management (DSM) algorithm (commonly referred to as 'Level 2') for many users sharing a common cable bundle. It is the only DSM algorithm that jointly address the problems of power allocation and subcarrier allocation in frequency division duplex (FDD) systems. The NRIA solves this optimization problem in a novel way by introducing new bitrate relations among the users. In [2] we showed that the NRIA achieves better bitrate performance than the IWFA and can achieve almost as good performance as the optimal spectrum balancing algorithm [4], but with much lower requirements on complexity.

The NRIA is based on the iterative water-filling algorithm (IWFA) [3] for finding an efficient power allocation, but extends the IWFA by finding the achievable bitrates of the IWFA and by optimizing the FDD bandplan for discrete multitone (DMT) systems. The result is high performance combined with low complexity, which is achieved with two types of useful parameters that well reflect many business scenarios: the desired network asymmetry and the desired user priorities.

In this paper we present the *constrained* normalized-rate iterative algorithm (C-NRIA), which extends the NRIA by ensuring fixed bitrates to certain users and assigning variable bitrates to the remaining users on a best effort basis. Both types of users (which we hence call fixed-rate and variable-rate users), however, are incorporated in the DSM optimization process by the C-NRIA in order to find an efficient FDD bandplan that is common for all users and an unique power allocation for each user in a cable bundle.

This work was partially financed by the Austrian K*plus* program.

Network service providers that offer DSL often face a difficult dilemma: how to efficiently balance the cable resources among the fixed-rate and variable-rate users in both transmission directions while preserving a number of desirable properties like certain priorities among the users and certain ratios between the downstream and upstream bitrates. Here, we address this high dimensional non-convex optimization problem and describe how it is possible to solve it efficiently with the C-NRIA. The solution is based on a key observation which describes in a simple way how the users' bitrates are related in a network. With this insight we show that it is sufficient to introduce only a single balancing parameter for the NRIA to split the capacity appropriately between the fixed-rate and the variable-rate users. The resulting C-NRIA searches for the appropriate value of this balancing parameter in order to find the desired operation point in a DSL network.

The paper is organized as follows: Section II briefly describes the basic NRIA; Section III formulates the optimization problem and derives the mathematical framework required to solve the problem; Section IV presents the C-NRIA, which efficiently solves the optimization problem; Section V presents some simulation results; and Section VI summarizes the paper.

II. Short review of the NRIA

The novel idea used for the NRIA is to set-up some relations among different bitrates rather than to rely on the achievable bitrates being known *a priori*, which is impractical. To establish these relations, the NRIA uses two types of parameters: an asymmetry parameter a and user priority values α.

The asymmetry parameter a specifies the ratio between the total desired downstream and upstream bitrates:

$$\sum_{u=1}^{U} R_{u,DS} = a \sum_{u=1}^{U} R_{u,US}, \tag{1}$$

where U denotes the number of users, $R_{u,DS}$ and $R_{u,US}$ denote the bitrates of user u in the downstream (from the network to the user) and upstream transmission directions, respectively. Based on the parameter a, the NRIA finds an FDD bandplan between the downstream and upstream directions which is common for all users. In practice, with a Zipper-DMT system

[5] the bandplan is achieved by assigning each individual subcarrier to one of the two transmission directions.

The priority values $\alpha_{u,dir}$ with $u \in \{1,\ldots,U\}$ and $dir \in \{DS, US\}$, are used to specify how much of the total cable capacity for each transmission direction is to be assigned to each user. The relations between the user bitrates are then expressed as

$$\frac{R_{1,dir}}{\alpha_{1,dir}} = \frac{R_{2,dir}}{\alpha_{2,dir}} = \ldots = \frac{R_{U,dir}}{\alpha_{U,dir}}, \text{ with } \sum_{u=1}^{U} \alpha_{u,dir} = 1. \quad (2)$$

Using these parameters it can be shown [6] that the downstream and upstream bitrates for each user are related by

$$R_{u,DS} = a \cdot \frac{\alpha_{u,DS}}{\alpha_{u,US}} \cdot R_{u,US}, \quad \text{for} \quad u = 1, 2, \ldots, U. \quad (3)$$

The number of bits that can be transmitted reliably by user u in one upstream DMT symbol is

$$R_{u,US} = \sum_{n \in I_{US}} \log_2 \left(1 + \frac{|H_{uu}^n|^2 P_u^n}{\Gamma N_u^n}\right), \quad (4)$$

where Γ is the SNR gap, which for a given symbol error rate and signal constellation represents the loss compared to the Shannon channel capacity; I_{US} is the set of subcarriers that are used in the upstream; H_{uu}^n, P_u^n, and N_u^n are the (direct) channel transfer function, the power, and the noise for user u on subcarrier n, respectively. The number of downstream bits $R_{u,DS}$, are derived correspondingly. The noise can be expressed as

$$N_u^n = \sum_{\substack{v=1 \\ v \neq u}}^{U} |H_{uv}^n|^2 P_v^n + P_V^n, \quad (5)$$

where H_{uv}^n denotes the far-end crosstalk (FEXT) channel transfer function from user v to user u; P_V^n denotes the power of the background noise on subcarrier n.

For predefined a, $\alpha_{u,DS}$ and $\alpha_{u,US}$, the NRIA finds jointly a bandplan, represented by I_{US} and I_{DS}, and power allocations, $P_{u,dir}^n$, for all users, $u \in \{1,\ldots,U\}$. The resulting bandplan and power allocations are based on the optimization

$$\text{maximize:} \quad \sum_{u=1}^{U} (R_{u,DS} + R_{u,US}),$$

$$\text{subject to:} \quad \begin{cases} \text{bitrate relations defined in (1) (2), and (3);} \\ \text{FDD transmission;} \\ \sum_{n=0}^{N-1} P_{u,dir}^n \leq P_{u,dir}^{\max}, \text{ for all users } u, \end{cases}$$

where $P_{u,dir}^{\max}$ denotes the total power constraint for user u on a specific transmission direction. The NRIA solves this high dimensional and non-convex optimization problem in a suboptimal way since the power allocations are based on the IWFA, and the FDD bandplan is based on an ad-hoc method [1], [2].

III. Problem Formulation and Mathematical Framework for the C-NRIA

The optimization problem we address and solve in this paper is similar to the problem given above with the additional constraint that some predefined fixed-bitrate users are to be assigned some selected fixed bitrates. Below we develop a mathematical framework to solve this optimization problem. Specifically, we will show that one parameter is sufficient to arbitrarily split the cable capacity among the fixed-rate and variable-rate users in both transmission directions. For the sake of simplicity we first develop this result for a single transmission direction and then extend it to both directions.

A. Single Transmission Direction

Assume that there are U^F fixed-rate users and U^V variable-rate users, where $U^F + U^V = U$. Let R_u^F, for $u = 1,\ldots,U^F$, and R_u^V, for $u = 1,\ldots,U^V$, denote the bitrates of the fixed-rate and variable-rate users, respectively. Furthermore, let T_u^F and T_u^V denote the corresponding target bitrates of the fixed-rate and variable-rate users, respectively. We assume that the selected target bitrates assigned to the fixed-rate users can always be supported, *i.e.*, $R_u^F \rightarrow T_u^F$. In Section IV we show how their maximum values can be determined.

Similarly to (2) the priority values and the bitrates of the fixed-rate and variable-rate users are related (in the selected direction) by

$$\underbrace{\frac{R_1^F}{\alpha_1^F} = \frac{R_2^F}{\alpha_2^F} = \ldots = \frac{R_{U^F}^F}{\alpha_{U^F}^F}}_{\text{fixed-rate users}} = \underbrace{\frac{R_1^V}{\alpha_1^V} = \frac{R_2^V}{\alpha_2^V} = \ldots = \frac{R_{U^V}^V}{\alpha_{U^V}^V}}_{\text{variable-rate users}}, \quad (6)$$

and

$$\sum_{u=1}^{U^F} \alpha_u^F + \sum_{u=1}^{U^V} \alpha_u^V = 1, \quad (7)$$

where α_u^F and α_u^V denote the priority values of the fixed-rate and variable-rate users, respectively. Note that the α_u^F values required to achieve the fixed bitrates are unknown in advance. However, by replacing R_u^F with T_u^F in (6) we can simply determine the relations among all of them.

For the right side in (6) neither R_1^V nor α_u^V is known in advance. But once again we can determine the relations among the α_u^V parameters by using the target rates for the variable-rate users, T_u^V. However, note that when the algorithm converges the bitrates assigned to the variable-rate users, R_u^V, can be smaller or larger than those targeted. Nevertheless, the relations in (6) still hold. The α_u^F and α_u^V should be selected such that (6) and (7) are fulfilled.

Example 1) We illustrate the selection of initial values for α_u^F and α_u^V for a network scenario with four users: two fixed-rate and two variable-rate users, thus, $U^F = 2$ and $U^V = 2$. We select $T_1^F = 20$ Mbit/s and $T_2^F = 10$ Mbit/s for the fixed-rate users. Suppose that we aim to offer the bitrates $T_1^V = 5$ Mbit/s and $T_2^V = 10$ Mbit/s to the variable-rate users (which may not be obtained). From these values and (6) we have these independent equations:

$$\frac{\alpha_1^F}{\alpha_2^F} = 2; \quad \frac{\alpha_1^F}{\alpha_1^V} = 4; \quad \text{and} \quad \frac{\alpha_1^F}{\alpha_2^V} = 2. \tag{8}$$

From (8) and (7): $\alpha_1^F = 4/9$, $\alpha_2^F = 2/9$, $\alpha_1^V = 1/9$, and $\alpha_2^V = 2/9$.□

Jointly increasing (or decreasing) the bitrates of one user group requires jointly decreasing (increasing) the bitrates of the other user group. The same holds for the user priority values. It is possible to search exhaustively for the priority values for which the fixed-rate users attain their target bitrates and which satisfy the relations defined in (6) under the constraint (7). However, this involves examining a large number of combinations. The following theorem is useful for developing a simple method to search for the desired user priority values.

Theorem 1: Given the bitrate relations (6) and constraint (7) *one* parameter is sufficient for properly balancing the bitrates of the fixed-rate and variable-rate users against each other.

Unfortunately, the proof of Theorem 1 is quite long and therefore it is not included in this paper. The interested reader can find it in [6].

The balance parameter, s, determines how the priority values between the fixed-rate and variable-rate users should be adjusted. The updated priority values for the fixed-rate users can be derived as

$$\widetilde{\alpha}_u^F = \alpha_u^F + \frac{s \cdot \alpha_u^F}{\sum\limits_{u=1}^{U^F} \alpha_u^F}, \quad \text{for} \quad u = 1, \ldots, U^F. \tag{9}$$

And similarly the updated priority values for the variable-rate users can be derived as

$$\widetilde{\alpha}_u^V = \alpha_u^V - \frac{s \cdot \alpha_u^V}{\sum\limits_{u=1}^{U^V} \alpha_u^V}, \quad \text{for} \quad u = 1, \ldots, U^F. \tag{10}$$

Furthermore, the search space for the balance parameter, s, must be kept within the interval $\left[-\sum_{u=1}^{U^F} \alpha_u^F, \sum_{u=1}^{U^V} \alpha_u^V\right]$. Thus, there exists an s within the given interval for which the fixed-rate users achieve their target bitrates. The two extreme cases are: 1) $s = -\sum_{u=1}^{U^F} \alpha_u^F$, which assigns the total cable capacity to the variable-rate users and 2) $s = \sum_{u=1}^{U^V} \alpha_u^V$, which assigns the total cable capacity to the fixed-rate users.

The task at hand is therefore to find the appropriate s, which gives the desired $\widetilde{\alpha}_u^F$ and $\widetilde{\alpha}_u^V$, satisfies (7) and grant the fixed-rate users their target bitrates. One efficient method goes like this: By using the initial values of α_u^F and α_u^V we calculate the supported bitrates of all users, which by definition satisfy (6). Thus, we need to check if these resulting supported bitrates of the fixed-rate users are larger, smaller, or equal to the corresponding target bitrates. Depending on the supported bitrate of a fixed-rate user, one of three cases can arise:

1) $R_u^F > T_u^F$; search among the negative values of s.
2) $R_u^F < T_u^F$; search among the positive values of s.
3) $R_u^F = T_u^F$; the initial user priority values are the correct ones, which results in $s = 0$.

These cases are illustrated in Figure 1. The bisection algorithm is used to search for s when case 1) or case 2) arises.

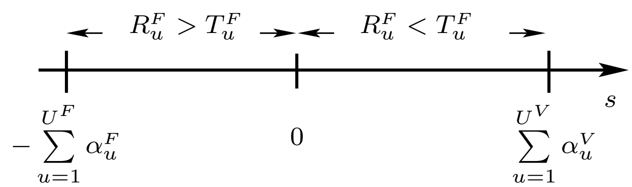

Fig. 1. Illustration of the search space for the balancing parameter s.

B. Bidirectional Transmission

DSL systems offer bidirectional transmission. Therefore, target bitrates for all users must be selected in both transmission directions. Specifically, the relations between downstream and upstream bitrates for the fixed-rate users are based on (3):

$$R_{u,DS}^F = a \cdot \frac{\alpha_{u,DS}^F}{\alpha_{u,US}^F} \cdot R_{u,US}^F, \quad \text{for} \quad u = 1, 2, \ldots, U^F. \tag{11}$$

That is, the downstream and upstream priority values for each user are related and cannot be selected arbitrarily. Given these relations we have:

$$a \cdot \frac{\alpha_{u,DS}^F}{\alpha_{u,US}^F} = c_u^F \left(= \frac{T_{u,DS}^F}{T_{u,US}^F} \right) \text{ for } u = 1, 2, \ldots, U^F, \tag{12}$$

where c_u^F is a constant. Relations corresponding to (11) and (12) apply also to the variable-rate users by replacing the 'F' superscripts with 'V'. In addition, (7) must be satisfied in both transmission directions.

To achieve all target bitrates for the fixed-rate users, which is the main goal, the relations in (6) must remain while searching for the priority values of the fixed-rate and variable-rate users. Note that this holds in both transmission directions. However, in general we cannot select new priority values for both user groups that simultaneously satisfy the relations (12) and (6). Instead, we calculate a new asymmetry value, $\widetilde{a}$, which maintains the desired bitrate relations. Thus, the new constraints for the fixed-rate users become:

$$\widetilde{a} \cdot \frac{\widetilde{\alpha}_{u,DS}^F}{\widetilde{\alpha}_{u,US}^F} = c_u^F \quad \text{for} \quad u = 1, 2, \ldots, U^F. \tag{13}$$

Corresponding constraints are calculated for the variable rate users.

Let us assume that we have found the updated $\widetilde{\alpha}_{u,DS}^F$ and $\widetilde{\alpha}_{u,DS}^V$ according to (9) and (10). Then the updated $\widetilde{\alpha}_{u,US}^F$ and $\widetilde{\alpha}_{u,US}^V$ can be derived from (13), after $\widetilde{a}$ is derived. The missing $\widetilde{a}$ can be found as follows: for the upstream transmission direction, based on (7) the new upstream priority values must fulfil

$$\widetilde{\alpha}_{1,US}^F + \ldots + \widetilde{\alpha}_{U^F,US}^F + \widetilde{\alpha}_{1,US}^V + \ldots + \widetilde{\alpha}_{U^V,US}^V = 1. \tag{14}$$

Substituting the upstream priority values of the fixed-rate and variable-rate users from (13) into (14) and solving for $\widetilde{a}$ gives:

$$\widetilde{a} = \frac{1}{\sum\limits_{u=1}^{U^F} \frac{\widetilde{\alpha}_{u,DS}^F}{c_u^F} + \sum\limits_{u=1}^{U^V} \frac{\widetilde{\alpha}_{u,DS}^V}{c_u^V}}. \tag{15}$$

Alternatively, it is also possible to derive the new asymmetry value $\widetilde{a}$ by using the upstream priority values.

IV. The Constrained NRIA

Based on the framework in the previous section the *constrained* NRIA (C-NRIA) can now be introduced in a straightforward manner. First, before running the C-NRIA, it is necessary to find out if the selected downstream and upstream bitrates to the fixed-rate users can be offered. This can be checked by running the basic NRIA with only the fixed-rate users included in the optimization process with the asymmetry parameter and user priority values calculated according to *Example 2* below. In this way, we can determine the maximum bitrates that can be offered to the fixed-rate users. Thus, if the desired bitrates for the fixed-rate users are smaller than the maximum bitrates found, the remaining cable resources will be utilized by the variable-rate users.

The pseudo-code of the C-NRIA is listed below as Algorithm 1. The algorithm operates as follows. For the selected target bitrates of all users (fixed-rate and variable-rate users), the C-NRIA calculates the corresponding asymmetry parameter as well as all the downstream and upstream priority values, by using (1) and (6). *Example 2* demonstrates this. With these initial priority and asymmetry values, the basic NRIA is then executed. In Algorithm 1 this corresponds to having $s = 0$ in the first iteration, *cf.* (9) and (10). In the following iterations we use the bisection method to search for the appropriate value of s. Depending on the supported bitrates by the fixed-rate users, one out of the three cases occurs as described in Section III-A, see also Fig. 1. When the target bitrates of the fixed-rate users are achieved, with some predefined accuracy, the desired solution is found and we stop the iterations.

Example 2) To illustrate how to calculate the maximum bitrates that can be offered to the fixed-rate users we first show how to calculate $\widetilde{a}$, $\widetilde{\alpha}_u^F$, and $\widetilde{\alpha}_u^V$ which satisfy the relations defined in (11) for both user groups.

Four users are assumed in this example – two fixed-rate and two variable-rate. Furthermore, assume we have selected the following target bitrates for the fixed-rate users: $T_{1,DS}^F = 33$ Mbit/s, $T_{2,DS}^F = 40$ Mbit/s, $T_{1,US}^F = 38$ Mbit/s, and $T_{2,US}^F = 47$ Mbit/s. We also aim to offer the variable-rate users the following bitrates: $T_{1,DS}^V = 15$ Mbit/s, $T_{2,DS}^V = 12$ Mbit/s, $T_{1,US}^V = 10$ Mbit/s, and $T_{2,US}^V = 5$ Mbit/s. However, those variable rates will not be guaranteed.

The asymmetry parameter is calculated as in (1); thus, $a = 1$. The downstream and upstream priority values of the fixed-rate and the variable-rate users are calculated as in *Example 1* in Section III-A. Table I summarizes the corresponding downstream and upstream priority values for the selected bitrates. As explained in Section III-A, we select $s = \alpha_{1,DS}^V + \alpha_{2,DS}^V = 0.27$, when we assign the total cable resources to the fixed-rate users (*i.e.*, one of the extreme cases). Substituting $s = 0.27$ into (9) and (10) we get: $\widetilde{\alpha}_{1,DS}^F = 0.45$, $\widetilde{\alpha}_{2,DS}^F = 0.55$, $\widetilde{\alpha}_{1,DS}^V = 0$, and $\widetilde{\alpha}_{2,DS}^V = 0$.

For $a = 1$ and the α_u^F and α_u^V values in Table I, we get the constants c_u^F and c_u^V by using (12); thus, $c_1^F = 0.87$, $c_2^F = 0.85$, $c_1^V = 1.50$, and $c_2^V = 2.40$. Substituting these c_u^F and c_u^V as well as $\widetilde{\alpha}_{u,DS}^F$ and $\widetilde{\alpha}_{u,DS}^V$ into (15) gives $\widetilde{a} = 0.86$. Using (13) we get: $\widetilde{\alpha}_{1,US}^F = 0.44$, $\widetilde{\alpha}_{2,US}^F = 0.56$, $\widetilde{\alpha}_{1,US}^V = 0$, and $\widetilde{\alpha}_{2,US}^V = 0$. For these calculated $\widetilde{a}$, $\widetilde{\alpha}_{DS}^F$, and $\widetilde{\alpha}_{US}^F$ values we run the NRIA and find the maximum bitrates, which can be supported in downstream and upstream for the fixed-rate users.□

TABLE I

Downstream (DS) and upstream (US) priority values for given bitrates of the fixed-rate (FR) and variable-rate (VR) users.

u	FR user bitrates (Mbit/s) DS	FR user bitrates (Mbit/s) US	VR user bitrates (Mbit/s) DS	VR user bitrates (Mbit/s) US	FR user priorities DS	FR user priorities US	VR user priorities DS	VR user priorities US
1	33.00	38.00	15.00	10.00	0.33	0.38	0.15	0.10
2	40.00	47.00	12.00	5.00	0.40	0.47	0.12	0.05
Σ	73.00	85.00	27.00	15.00	0.73	0.85	0.27	0.15

Algorithm 1: The constrained NRIA

Initialize:
T_{DS}^F, T_{US}^F {Mandated bitrates for the fixed-rate users}
T_{DS}^V, T_{US}^V {Desired bitrates for the variable-rate users}

From: T_{DS}^F, T_{US}^F and T_{DS}^V, T_{US}^V
calculate: $\alpha_{DS}^F,\ \alpha_{DS}^V,\ \alpha_{US}^F,\ \alpha_{US}^V$; using (6) and (7)
$s_{min} = -\sum_{u=1}^{U^F} \alpha_u^F$; $s_{max} = \sum_{u=1}^{U^V} \alpha_u^V$
$s = 0$
repeat
  For s calculate: $\widetilde{a}, \widetilde{\alpha}_{DS}^F, \widetilde{\alpha}_{DS}^V, \widetilde{\alpha}_{US}^F, \widetilde{\alpha}_{US}^V$; using (9), (10), (13), and (15)
  $[R_{DS}^F, R_{US}^F, R_{DS}^V, R_{US}^V]$ = **NRIA**$(\widetilde{a}, \widetilde{\alpha}_{DS}^F, \widetilde{\alpha}_{DS}^V, \widetilde{\alpha}_{US}^F, \widetilde{\alpha}_{US}^V)$
  if $R_{1,DS}^F > T_{1,DS}^F$ **then**
    $s_{max} = s$
  else
    $s_{min} = s$
  end if
  $s = \frac{s_{min}+s_{max}}{2}$
until R_{DS}^F, R_{US}^F approach T_{DS}^F, T_{US}^F with some desired accuracy

In the following we assume that the selected target bitrates of the fixed-rate users are always supported. This can be verified by running the NRIA with the calculated asymmetry value and the user priority values as explained in *Example 2*.

V. Simulation Results

For all simulations we use the network scenario shown in Figure 2, with two fixed-rate and two variables-rate users. The main simulation parameters are based on the ETSI VDSL DMT-based standard [7]. Thus, the maximum total power for each user and each transmission direction is set to 11.5 dBm. Furthermore, we assumed a DMT system with 2048 subcarriers. The cable model used is the so called "TP100" [7], which has 0.5 mm conductors. The FEXT coupling model used is specified in [7].

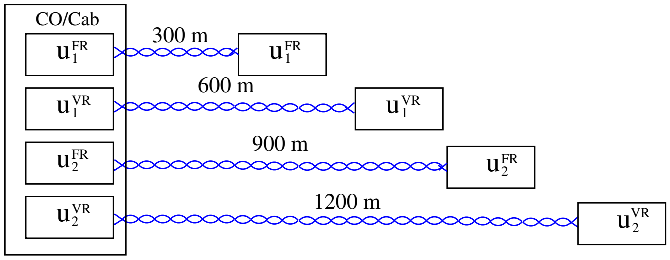

Fig. 2. A network scenario with four users, two fixed-rate and two variable-rate, which are deployed from the central office (CO) or a cabinet (Cab).

Example 3) Assume that we have selected the bitrates given in Table I as the target bitrates for the fixed-rate users and as the desired bitrates for the variable-rate users. Thus, the same priority values given in Table I are assigned to all users and the asymmetry parameter is $a = 1$.

First we should confirm that the selected target bitrates to the fixed-rate users can be offered at all. As explained in Section IV, we verify this by running the basic NRIA with only the fixed-rate users included in the optimization process. Because we have used the bitrates and priority values in Table I we run the NRIA with the following values: $\widetilde{a} = 0.86$, $\widetilde{\alpha}^F_{1,DS} = 0.45$, $\widetilde{\alpha}^F_{2,DS} = 0.55$, $\widetilde{\alpha}^F_{1,US} = 0.44$, $\widetilde{\alpha}^F_{2,US} = 0.56$, which have been calculated in *Example 2*. The NRIA finds that the maximum bitrates that can be supported by the fixed-rate users are $R^{F,\max}_{1,DS} = 36.49$ Mbit/s, $R^{F,\max}_{2,DS} = 44.06$ Mbit/s, $R^{F,\max}_{1,US} = 41.77$ Mbit/s, and $R^{F,\max}_{2,US} = 51.47$ Mbit/s. Thus, the selected target bitrates can be offered to the fixed-rate users.

We then run the C-NRIA with the targets from *Example 2*. The supported bitrates and the calculated priority values of all users are summarized in Table II. By comparing the results in Table II and Table I we conclude that we can guarantee the fixed-rate users the selected target bitrates and increase the bitrates of the variable-rate users by approximately 77% compared to the bitrates we first aimed to offer them.□

Example 4) We now change the downstream target bitrate of the first fixed-rate user to $T^F_{1,DS} = 36$ Mbit/s. Also for this example we initialize the priority values for all users to those shown in Table I. Thus, the new target bitrates of the other fixed-rate users are $T^F_{2,DS} = 43.63$ Mbit/s, $T^F_{1,US} = 41.45$ Mbit/s, and $T^F_{2,US} = 51.27$ Mbit/s. The supported bitrates and the calculated priority values of all users are summarized in Table III. We conclude that to guarantee the fixed-rate users the selected target bitrates we should reduce the bitrates of the variable-rate users by approximately 33% compared to the bitrates we had aimed to offer them.□

TABLE II
THE SUPPORTED BITRATES AND THE CALCULATED PRIORITY VALUES OF ALL USERS FOR THE SCENARIO IN FIG. 2 FOR $T^F_{1,DS} = 33$ MBIT/S .

	FR user bitrates (Mbit/s)		VR user bitrates (Mbit/s)		FR user priorities		VR user priorities	
u	DS	US	DS	US	DS	US	DS	US
1	32.99	38.06	26.52	17.71	0.27	0.34	0.22	0.16
2	39.98	47.07	21.22	8.95	0.33	0.42	0.18	0.08
Σ	72.97	85.13	47.74	26.66	0.60	0.76	0.40	0.24

TABLE III
THE SUPPORTED BITRATES AND THE CALCULATED PRIORITY VALUES OF ALL USERS FOR THE SCENARIO IN FIG. 2 FOR $T^F_{1,DS} = 36$ MBIT/S.

	FR user bitrates (Mbit/s)		VR user bitrates (Mbit/s)		FR user priorities		VR user priorities	
u	DS	US	DS	US	DS	US	DS	US
1	36.00	41.54	10.08	6.73	0.37	0.40	0.10	0.07
2	43.63	51.34	8.06	3.36	0.45	0.50	0.08	0.03
Σ	79.63	92.88	18.14	10.09	0.82	0.90	0.18	0.10

VI. CONCLUSION

In this paper we presented the constrained normalized-rate iterative algorithm (C-NRIA). The C-NRIA is an extension of the NRIA [1], [2] for solving the dynamic spectrum management (DSM) problem for which the bitrates of some users need to be guaranteed and therefore fixed in advance. This reflects many important business scenarios where a number of customers in a network must be guaranteed a certain DSL service by the operator, while the remaining customers can be offered a best effort service.

We showed that a single balancing parameter is sufficient to split the cable resources among the fixed-rate and variable-rate users in both transmission directions. Furthermore, we presented an efficient method to search for the desired balancing parameter based on the bisection algorithm. With simulation examples we showed how the C-NRIA can split the cable capacity between fixed-rate and variable-rate users in an efficient way.

ACKNOWLEDGMENT

We would like to thank Prof. Hans Weinrichter and Prof. Per Ödling for their valuable ideas and support.

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