Verification of Multipair Copper-Cable Model by Measurements

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***Abstract*—Analysis, assessment, and design of advanced wireline transmission schemes over multipair copper cables require accurate knowledge of the channel properties. This paper investigates modeling of multiconductor cables based on interpair impedance measurements. A unified approach to the application of the "Cioffi" model is introduced. The direct measurement approach of the underlying interpair impedances yields a good match with the alternative approach suggested in the recent study of Cioffi *et al.* Crosstalk coupling functions derived from the model exhibit a good match with the corresponding direct measurements in the case in which the modeled length is close to the length of the interpair impedance measurements. However, the prediction power of this model with respect to termination impedance is limited.**

***Index Terms*—Cable measurements, channel modeling, digital subscriber line (DSL), multiconductor model, multiple-input multiple-output (MIMO).**

I. Introduction

HIGH-SPEED Internet access over telephone lines—unshielded twisted pairs of copper—keeps gaining importance. Various digital subscriber line technologies [3], [4] enable services with different data rates, depending on the loop length and on the noise environment. The continuously growing demand for data rate, as well as the operators' goal of exploiting their cables in the best possible way, drives the development of wireline communications. Next-generation techniques, like vectored methods [5] and multiuser schemes [6] proposed in the context of dynamic spectrum management [7], view the cable as a multiple-input multiple-output (MIMO) channel. The investigation of these techniques requires accurate knowledge of the channel properties.

The two favored MIMO cable models [1], [8] result in equivalent ABCD-matrix descriptions. However, the underlying parameters and measurements required for their determination differ. Measurements and modeling results based on the "Joffe" model [8] are presented in [9]. The focus of this work is on the "Cioffi" model [1], which requires interpair impedance measurements, i.e., measurements of the impedances between wires of different pairs in a cable. Given this model and an arbitrary termination of the cable at each side, it is possible to determine crosstalk coupling functions, insertion-loss functions, transfer functions, and input impedances. It should be pointed out that the advantage of this multiconductor modeling approach, compared to direct measurements of individual coupling or transfer functions, does not lie in reduced measurement effort but, at least theoretically, rather in gaining independence of termination impedances and loop length.

II. MIMO Cable Modeling

A cable of length L with m pairs is described by its characteristic impedance matrix $\boldsymbol{Z}_{\mathrm{c}} \in \mathbb{C}^{n\times n}$ and its propagation matrix $\gamma \in \mathbb{C}^{n\times n}$, where $n = 2m - 1$ [1], [10]. On each side of the cable, n voltages $V_k(f)$, $1 \le k \le n$ and n currents $I_k(f)$, $1 \le k \le n$ can be defined as shown for the two-pair case ($m = 2$, $n = 3$) in Fig. 1. All currents, voltages, and impedances depend on the frequency f, which we will omit in the following wherever possible, for the sake of simple notation. The voltages $\boldsymbol{V} = [V_n \cdots V_1]^{\mathrm{T}}$ and the currents $\boldsymbol{I} = [I_n \cdots I_1]^{\mathrm{T}}$ fulfill $\boldsymbol{V} = \boldsymbol{Z}_{\mathrm{in}}\boldsymbol{I}$, where

$$\boldsymbol{Z}_{\mathrm{in}} = \boldsymbol{Z}_{\mathrm{c}}\left(\cosh(\boldsymbol{\gamma}^{\mathrm{T}}L) + \sinh(\boldsymbol{\gamma}^{\mathrm{T}}L)\boldsymbol{Z}_{\mathrm{t}}^{-1}\boldsymbol{Z}_{\mathrm{c}}\right) \times \left(\cosh(\boldsymbol{\gamma}^{\mathrm{T}}L)\boldsymbol{Z}_{\mathrm{t}}^{-1}\boldsymbol{Z}_{\mathrm{c}} + \sinh(\boldsymbol{\gamma}^{\mathrm{T}}L)\right)^{-1} \quad (1)$$

is the input impedance matrix[1] and $\boldsymbol{Z}_{\mathrm{t}}$ is the far-end termination impedance matrix, for which $\boldsymbol{V}' = \boldsymbol{Z}_{\mathrm{t}}\boldsymbol{I}'$ holds with $\boldsymbol{V}' = [V_n' \cdots V_1']^{\mathrm{T}}$ and $\boldsymbol{I}' = [I_n' \cdots I_1']^{\mathrm{T}}$. The voltages $\boldsymbol{V}$ and $\boldsymbol{V}' = \boldsymbol{T}\boldsymbol{V}$ are related via the transfer-function matrix

$$\boldsymbol{T} = \left(\cosh(\boldsymbol{\gamma}^{\mathrm{T}}L) + \sinh(\boldsymbol{\gamma}^{\mathrm{T}}L)\boldsymbol{Z}_{\mathrm{t}}^{-1}\boldsymbol{Z}_{\mathrm{c}}\right)^{-1}. \quad (2)$$

For a given cable, described by $\boldsymbol{Z}_{\mathrm{c}}$ and $\boldsymbol{\gamma}$, and for a given setup, which consists of the excitation of a certain pair and the termination of all remaining ends, important characteristics can be determined from the voltages $\boldsymbol{V}$ and $\boldsymbol{V}'$. For example, in the setup shown in Fig. 1, insertion loss $H_{\mathrm{ins}} = V_1'/V_1$, transfer function $H_{\mathrm{trans}} = V_1'/V_{\mathrm{s}}$, near-end crosstalk (NEXT) coupling

[1]Note that all operations like $\cosh(\cdot)$, $\sinh(\cdot)$, and $(\cdot)^{-1/2}$ with matrix arguments denote matrix functions.

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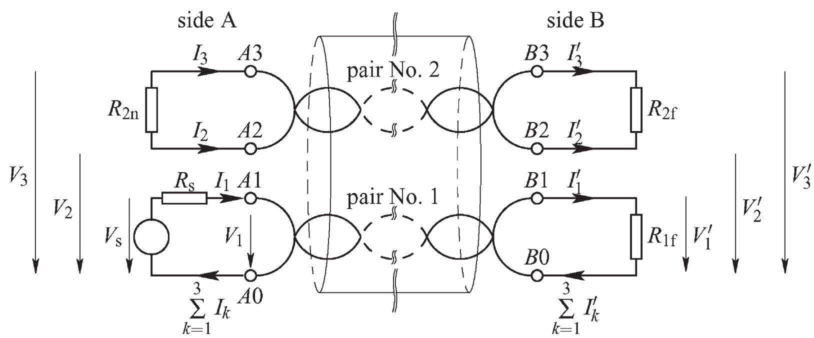

Fig. 1. Simple example of a multipair cable channel with two pairs, excitation of pair No. 1 by V_s, and termination of the remaining ports by R_{1f}, R_{2n}, and R_{2f}.

function $H_{\text{next}} = (V_3 - V_2)/V_1$, and far-end crosstalk (FEXT) coupling function $H_{\text{fext}} = (V'_3 - V'_2)/V_1$ are typical channel properties of interest from a communication point of view. A unified approach to finding $\boldsymbol{V}$ involves solving a system of $2n$ linear equations. The unknowns are $\boldsymbol{V}$ and $\boldsymbol{I}$. The equations are obtained from the conditions imposed by the near-end termination of the wire ends (side A) and by the excitation. For example, the equation system for the setup shown in Fig. 1 is given by

$$\begin{bmatrix} Z_{\text{in},33} & Z_{\text{in},32} & Z_{\text{in},31} & -1 & 0 & 0 \\ Z_{\text{in},23} & Z_{\text{in},22} & Z_{\text{in},21} & 0 & -1 & 0 \\ Z_{\text{in},13} & Z_{\text{in},12} & Z_{\text{in},11} & 0 & 0 & -1 \\ R_{2n} & 0 & 0 & 1 & -1 & 0 \\ 0 & R_{2n} & 0 & 1 & -1 & 0 \\ 0 & 0 & R_s & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{V} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{bmatrix} \tag{3}$$

where $Z_{\text{in},ij}$ denotes the element in row $n-i+1$ and column $n-j+1$ of the input impedance matrix $\boldsymbol{Z}_{\text{in}}$ given by (1). Solving the equation system yields $\boldsymbol{V}$ and, thus, $\boldsymbol{V}' = \boldsymbol{T}\boldsymbol{V}$ as well, with $\boldsymbol{T}$ given by (2). The far-end termination (side B), specified by $\boldsymbol{Z}_t$, is required for computing both $\boldsymbol{Z}_{\text{in}}$ and $\boldsymbol{T}$. For our example, $\boldsymbol{Z}_t$ is given by

$$\boldsymbol{Z}_t^{-1} = \begin{bmatrix} 1/R_{2f} & -1/R_{2f} & 0 \\ -1/R_{2f} & 1/R_{2f} & 0 \\ 0 & 0 & 1/R_{1f} \end{bmatrix}.$$

Note that all variables are frequency-dependent.

III. Interpair Measurements

The characteristic impedance matrix $\boldsymbol{Z}_c$ and the propagation matrix $\boldsymbol{\gamma}$ of an L_{meas}-meter-long cable are given by

$$\boldsymbol{Z}_c = \left(\boldsymbol{Z}_s \boldsymbol{Z}_o^{-1}\right)^{-1/2} \boldsymbol{Z}_s$$

and

$$\boldsymbol{\gamma} = \frac{1}{L_{\text{meas}}} \tanh^{-1}\left(\left(\boldsymbol{Z}_s \boldsymbol{Z}_o^{-1}\right)^{-1/2}\right)$$

respectively, where $\boldsymbol{Z}_o$ is the open-circuit impedance matrix, and $\boldsymbol{Z}_s$ is the short-circuit impedance matrix [1]. The element in row $n-i+1$ and column $n-j+1$ of $\boldsymbol{Z}_o$ and $\boldsymbol{Z}_s$ is defined as

$$Z_{\{o,s\},n-i+1,n-j+1} = \begin{cases} \left.\frac{V_i}{I_i}\right|_{I_k=0, \quad k\in\{1,\ldots,N\}\backslash i}, & i=j \\ \left.\frac{V_{\max\{i,j\}}-V_{\min\{i,j\}}}{I_{\max\{i,j\}}}\right|_{I_k=0, \quad k\in\{1,\ldots,N\}\backslash\{i,j\}}, & i\neq j. \end{cases} \tag{4}$$

For the short-circuit measurement (Z_s), all wires on side B are connected ($V'_k = 0$, $1 \le k \le n$), and for the open-circuit measurement (Z_o), all wires on side B are left open ($I'_k = 0$, $1 \le k \le n$). Note that the elements off the main diagonal $(i \neq j)$ cannot be measured directly using an impedance analyzer. For example, the definition of $Z_{12} = (V_2 - V_1)/I_2|_{I_3=0}$ suggests to leave A3 open but does not imply that $I_2 = -I_1$, which would allow measuring the impedance between A2 and A1. Therefore, we propose to measure

$$M_{ij} = \begin{cases} \left.\frac{V_i}{I_i}\right|_{I_k=0, \quad k\in\{1,\ldots,N\}\backslash\{i\}}, & i=j \\ \left.\frac{V_i-V_j}{I_i}\right|_{I_k=0, \quad k\in\{1,\ldots,N\}\backslash\{i,j\}; \ \sum_{1\le k\le N} I_k=0}, & i\neq j \end{cases} \tag{5}$$

for $1 \le j < i \le N$ and calculate the impedance matrix entries according to

$$Z_{ij} = Z_{ji} = \begin{cases} M_{ii}, & i=j \\ \frac{1}{2}(Z_{ii} + Z_{jj} - M_{ij}), & i \neq j \end{cases} \tag{6}$$

for $1 \le j < i \le N$. Note that M_{ij} can be conveniently measured using an impedance analyzer.

As an example, Fig. 2 shows the measured $Z_{s,12}$ of a 0.6-mm cable with six pairs (vendor identification: F02YHJ2Y, PMD6 × 2 × 0.6 [11]) of length $L_{\text{meas}} = 100$ m. We compare the result of our direct impedance measurement (solid line) with the result obtained following an alternative approach (dashed line) suggested in [2, p. 2], which involves measuring the voltage ratio[2] VG5 $= V_2/(V_1 - V_3)|_{I_1=I_3=0}$. The two approaches yield almost perfectly matching results (they are

[2]The terminology VG5 is adopted from [2].

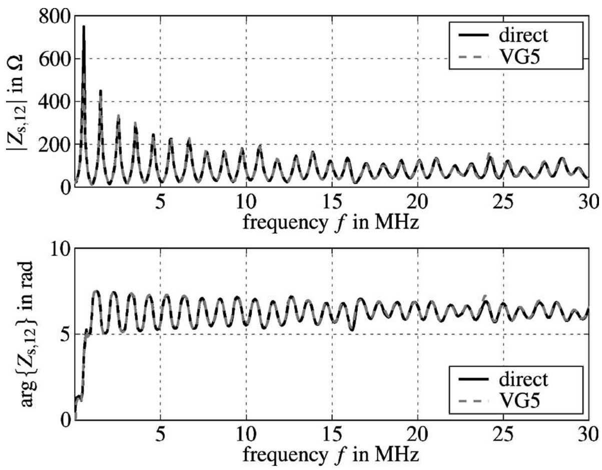

Fig. 2. Measured $Z_{s,12}$ of a 0.6-mm cable with six pairs (vendor identification: F02YHJ2Y, PMD6 × 2 × 0.6) of length $L_{\mathrm{meas}} = 100$ m. The deviation of the direct impedance measurement according to (5) and (6) (solid line) from the result obtained by an alternative approach (see [2, p. 2]), (dashed line) is negligible.

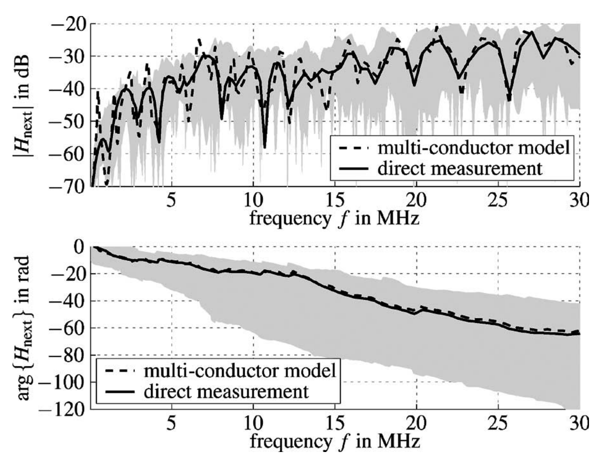

Fig. 3. Comparison of NEXT coupling function $H_{\mathrm{next}} = (V_3 - V_2)/V_1$ for the setup shown in Fig. 1, with $L = 100$ m and $R_{1f} = R_{2n} = R_{2f} = 135\,\Omega$, which is derived from the multiconductor model (dashed line) with direct measurement (solid line). The shaded area marks the spread of all 30 directly measured NEXT coupling functions.

indistinguishable in the figure); however, the determination of VG5 is more intricate since it involves voltage measurements using a high-impedance probe.

IV. Comparison of Modeled Properties With Direct Measurements

The interpair impedance measurements described in the previous section yield the cable parameters $\boldsymbol{Z}_{\mathrm{c}}$ and $\boldsymbol{\gamma}$, which allow the derivation of various cable properties, as described in Section II. Fig. 3 shows the NEXT coupling function $H_{\mathrm{next}} = (V_3 - V_2)/V_1$ for $L = 100$ m and $R_{1f} = R_{2n} = R_{2f} = 135\,\Omega$. The modeled coupling function (dashed line) based on the impedance measurements for $L_{\mathrm{meas}} = 100$ m matches the direct measurement (solid line) reasonably well, considering the fact that the cable parameterization involves 12 impedance measurements. The shaded area marks the spread of all 30 directly measured NEXT coupling functions.

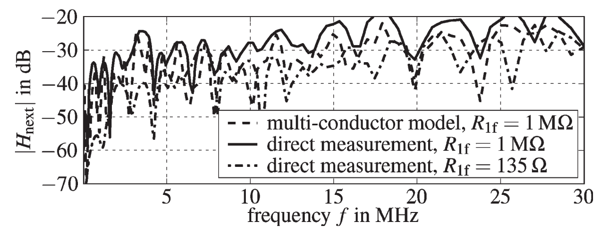

Fig. 4. Comparison of NEXT coupling functions derived from the model based on impedance measurements ($L_{\mathrm{meas}} = 100$ m) for $L = 100$ m, $R_{1f} = 1$ MΩ, and $R_{2n} = R_{2f} = 135\ \Omega$ (dashed line) with direct measurement (solid line). In order to assess the degree of deviation, the direct measurement for $R_{1f} = 135\ \Omega$, $R_{2n} = R_{2f} = 135\ \Omega$, and $L = 100$ m is shown (dashed–dotted).

Fig. 4 shows the prediction power of the model based on the impedance measurements for $L_{\mathrm{meas}} = 100$ m with respect to termination impedance. The plot shows the NEXT coupling functions for $L = 100$ m and $R_{1f} = 1$ MΩ, $R_{2n} = R_{2f} = 135\ \Omega$ derived from the multiconductor model (dashed line) and measured directly (solid line). The variation with respect to the directly measured coupling function for $R_{1f} = 135\ \Omega$ (dashed–dotted line), which is plotted as a reference, is considerable.

V. Discussion of Results and Conclusion

The match of modeled and measured coupling functions is good in the case in which the modeled cable length L is equal to the interpair impedance measurement length L_{meas}. However, apart from roughly following the same trend, the predicted coupling function for an arbitrary termination exhibits a poor match with its directly measured counterpart for the following reasons. First, the model is based on results of 12 (for the two-pair case) individual measurements, which makes the model sensitive to deviations. Second, and more importantly, the model assumes that the characteristic matrix impedance $\boldsymbol{Z}_{\mathrm{c}}$ and the propagation matrix $\boldsymbol{\gamma}$ are independent of the length, i.e., the cable properties are uniform over the length. Such an assumption is not justified for many conductors in a bundle cable. Consequently, the prediction power of the model with respect to length is limited. An improved modeling approach would probably involve modeling of very short cable segments and integration over these segments in order to obtain the properties for a certain length.

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