SPLITTING THE LEAST-MEAN-SQUARE ALGORITHM

*Thomas Magesacher**, *Per Ola Börjesson*†, *Per Ödling*‡, *and Tomas Nordström*§

Telecommunications Research Center Vienna (ftw.)
Donau-City Straße 1/III
A-1220 Vienna, Austria
magesacher@ftw.at

ABSTRACT

The least-mean-square (LMS) algorithm is an adaptation scheme widely used in practice due to its simplicity. In some applications the involved signals are continuous-time. Then, usually either a fully analog implementation of the LMS algorithm is applied or the input data are sampled by analog-to-digital (AD) converters to be processed digitally. A purely digital realization is most often the preferred choice, however, it becomes costly for high-frequency input signals since fast AD converters are needed.

In this paper we propose a hybrid analog/digital approach allowing the AD conversion rate to be as low as the update-rate of the LMS algorithm. We demonstrate the advantage of this approach applying it to an interference cancellation problem occurring in wireline communications: the sampling rate of the AD converters is reduced by a factor of 250.

1. INTRODUCTION

Most often adaptive algorithms are discussed in their discrete-time versions in literature, see *e.g.*, [1]. However, since the "real world" is analog, there are many cases when the observed data are continuous-time and often also of high frequency. A particularly low-complexity adaptation scheme is the least-mean-square (LMS) algorithm, an important member of the family of stochastic gradient algorithms.

All-analog implementations of the LMS algorithm are feasible in practice, as demonstrated, *e.g.*, in [2],[3]. However, hybrid analog/digital signal processing is known to have the potential of combining the best of both the analog and the digital world [4]. Analog hardware can handle high-frequency signals more efficiently, but is limited mostly to simple and preferably linear operations. Digital signal processing can easily deal with nonlinear operations but is limited to relatively low operating rates.

In this paper we focus on the LMS algorithm applied to a generic model: the multiple-input adaptive linear combiner, operating on continuous-time input data, cf. Fig. 1. We split the LMS algorithm into an analog and a digital part, an idea which is applied to the recursive least-squares (RLS) algorithm in [5]. In Section 2, the derivation of the LMS algorithm is modified accordingly. Section 3 presents a frequency-domain interpretation. In Section 4, we demonstrate the feasibility of our approach applying it to a problem encountered in wireline communications.

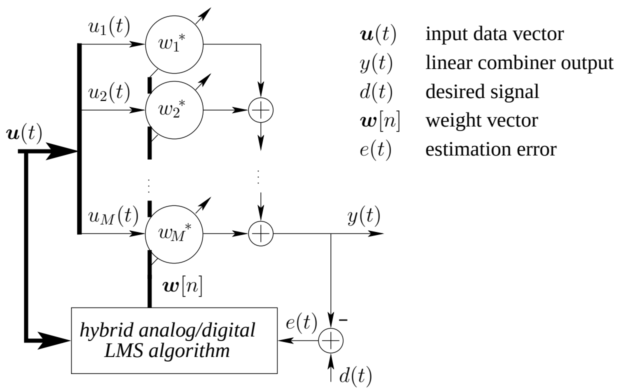

Figure 1: Continuous-time multiple-input adaptive linear combiner. The M weights are updated by a hybrid analog/digital version of the LMS algorithm illustrated in Fig. 2.

---

* Thomas Magesacher is with the Telecommunications Research Center Vienna, Austria. He is also a Ph.D. candidate at the Institute for Integrated Circuits at Munich Technical University, Germany.

† Dr. Per Ola Börjesson is a Professor of Signal Processing at the Department of Electroscience, Lund Institute of Technology, Sweden.

‡ Docent Dr. Per Ödling is an Associate Professor at the Department of Electroscience, Lund Institute of Technology, Sweden.

§ Dr. Tomas Nordström is a Senior Researcher with the Telecommunications Research Center Vienna, Austria.

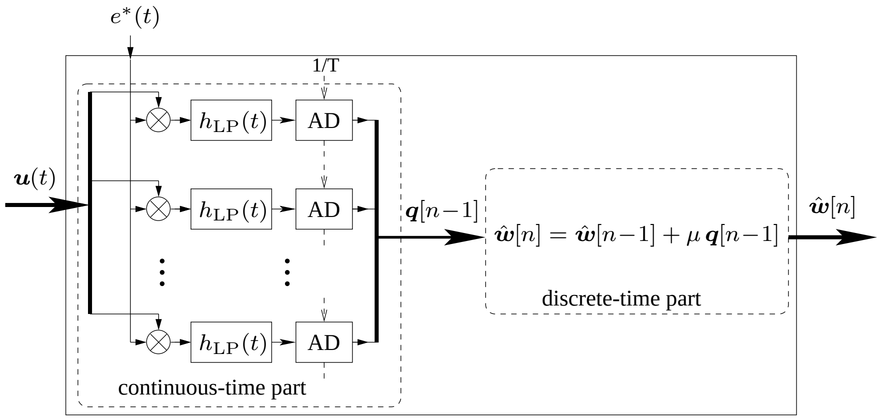

Figure 2: Block diagram of the hybrid analog/digital LMS algorithm adapting the coefficients of a linear combiner.

2. LMS ADAPTATION OF THE CONTINUOUS-TIME LINEAR COMBINER

Fig. 1 shows the continuous-time multiple-input linear combiner [6] adapted by an LMS algorithm. The update of the M weights is performed digitally, while the combiner part is processed in analog domain. Thus, the weights $\boldsymbol{w}(t)$ are piece-wise constant functions. Using the notation introduced in Fig. 1 and following [1], we base the derivation of the modified LMS algorithm on the method of steepest descent, formulated as

$$\boldsymbol{w}[n] = \boldsymbol{w}[n-1] + \mu(\boldsymbol{p} - \boldsymbol{R}\,\boldsymbol{w}[n-1]), \qquad (1)$$

where the positive constant μ is the step size of the LMS algorithm. The correlation functions of the stationary continuous parameter processes $u_i(t)$ and $d(t)$ evaluated in $\tau = 0$ are collected in

$$\boldsymbol{p} = \mathsf{E}\{\boldsymbol{u}(t+\tau)d^*(t)\}|_{\tau=0}, \qquad (2)$$

and the correlation functions of all the pairs of input processes $u_i(t)$ and $u_j(t)$, also evaluated in $\tau = 0$, form the matrix

$$\boldsymbol{R} = \mathsf{E}\{\boldsymbol{u}(t+\tau)\boldsymbol{u}^{\mathrm{H}}(t)\}|_{\tau=0}. \qquad (3)$$

Replacing $\boldsymbol{p}$ and $\boldsymbol{R}$ in (1) by their estimators

$$\hat{\boldsymbol{p}}(t_1) = \int_{t=-\infty}^{t=+\infty} \boldsymbol{u}(t)d(t)h_{\mathrm{LP}}(t_1 - t)dt \qquad (4)$$

and

$$\hat{\boldsymbol{R}}(t_1) = \int_{t=-\infty}^{t=+\infty} \boldsymbol{u}(t)\boldsymbol{u}^{\mathrm{H}}(t)h_{\mathrm{LP}}(t_1 - t)dt, \qquad (5)$$

we obtain for $t_1 = (n-1)T$ the recursive update rule for the weight vector:

$$\hat{\boldsymbol{w}}[n] = \hat{\boldsymbol{w}}[n-1] + \mu\boldsymbol{q}[n-1]. \qquad (6)$$

To fit the frequency-domain interpretation below, we refer to $\boldsymbol{q}[n]$ as the baseband error, given by

$$\boldsymbol{q}[n] = (\boldsymbol{u}(nT)\, e^*(nT)) \star h_{\mathrm{LP}}(nT), \qquad (7)$$

where $h_{\mathrm{LP}}(t)$ is the impulse response of a lowpass filter, whose design will be discussed below, and $\star$ is the convolution operator. The estimation error is $e(t) = d(t) - y(t)$ and the output of the linear combiner is given by

$$y(t) = \hat{\boldsymbol{w}}^{\mathrm{H}}[n]\boldsymbol{u}(t), \quad nT \le t < (n+1)T. \qquad (8)$$

Equations (6) and (7) represent the discrete-time and the continuous time part of the hybrid LMS algorithm, respectively. The block diagram is shown in Fig. 2.

3. FREQUENCY DOMAIN CONSIDERATIONS

A frequency-domain interpretation of the approach is depicted in Fig. 3. For this principle consideration, we choose the input signals $\boldsymbol{u}(t)$ and the desired signal $d(t)$ to be of bandpass nature with center frequency f_c. Thus also the error $e(t)$ is a bandpass signal. The product of input and error signals, formed by the continuous-time part of the algorithm, consists of spectral components around DC (f=0) and twice the center frequency f_c. The lowpass filter, characterized by its impulse response $h_{\mathrm{LP}}(t)$, eliminates the high-frequency components and thus serves as an anti-aliasing filter for the AD converter. Note that the sampling rate of the AD converter equals the update

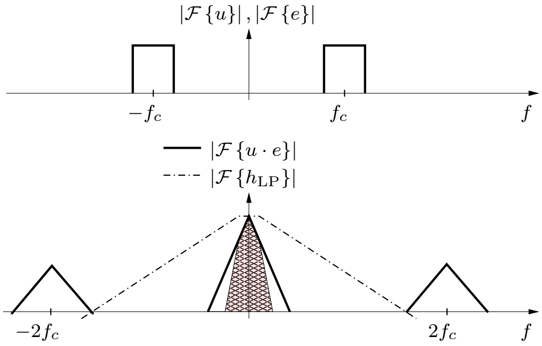

Figure 3: Fourier transforms $\mathcal{F}\{\cdot\}$ of the input signal, the error signal and their products: after multiplication of the signal, the algorithm processes only the lowpass-filtered part (shaded) of the product $u \cdot e$.

rate of the LMS algorithm and can be chosen independently from the bandwidth and the center frequency f_c of the signal.

The expectation of the filter output is given by

$$\mathsf{E}\{\boldsymbol{q}(t)\} = \underbrace{\int_{\tau=-\infty}^{\tau=+\infty} h_{\mathrm{LP}}(\tau)\ d\tau}_{\mathcal{F}\{h_{\mathrm{LP}}(t)\}|_{f=0}=H_{\mathrm{LP}}(0)} \quad \mathsf{E}\{\boldsymbol{u}(t)e^*(t)\}, \qquad (9)$$

where $H_{\mathrm{LP}}(0)$ is the DC gain of the lowpass filter. Since the bandwidth of the lowpass filter does not influence the expected value of the filtered signal, in principle an arbitrarily small bandwidth can be chosen. A limit is given by the latency of the filter, which rises with decreasing bandwidth and can cause instability of the algorithm. This is easily explained by viewing the filter as a delay in the loop. If the baseband error $\boldsymbol{q}[n]$ is delayed too much, it can be in counter-phase with respect to the weights. The resulting positive feedback would then cause instability of the algorithm.

4. APPLICATION EXAMPLE: NARROWBAND INTERFERENCE CANCELLATION

The elimination of a narrowband interference is a common problem in communications. In case a reference signal, correlated with the disturbance, is available, cancellation is an efficient solution [7].

4.1. Problem Background and Goal

Our example-application is based on a problem known as RFI (radio frequency interference) in wireline communications. Amateur radio (HAM) transmitters and broadcast radio stations generate narrowband interference which is picked up by the wires and added to the data signal. The power of the interference may easily be much larger than the power of the data signal, and thus cause an overload of the AD converter in the receiver [8].

We take a brief look at the physical layer of wireline communications in order to understand the origin of a signal that may serve as a reference employed in a cancellation approach to this problem. In principle, data transmission is accomplished by sending and receiving a differential-mode (DM) signal, which corresponds to the voltage difference between the wires of a loop. The common-mode (CM) signal, the arithmetic mean of the voltages measured between each wire and ground, is correlated with the DM signal, and contains, apart from noise, basically two components: a part correlated with the signal and one part correlated with the interference. A measure that quantifies the quality of a cable with respect to CM-to-DM conversion and vice versa is the balance of a cable defined in [9],[10]. As measurements have shown [11],[12], the interference part is, depending on the cable, often much stronger than the DM-signal part, thus the CM signal may serve as a reference for cancellation.

The purpose of the interference canceller is to protect the AD converter in the receiver from overloading. This results in two requirements. Firstly, convergence in the mean square of the LMS algorithm should be reached before the interference generated by an amateur radio transmitter that is turned on at a certain point in time reaches its maximum. In practice, this ramp-up time is around 1 ms. Secondly, a steady state suppression of the interference of around 10 dB to 20 dB is desirable in order to avoid an overload condition. The residual interference can, as long as it is low enough to pass the AD converter, be tackled by several different methods in the digital domain [13].

4.2. Canceller Structure

If we let the bandwidth of the interference go to zero, the reference signal is just a time-shifted and scaled version of the disturbance. We split the reference signal into two orthogonal components $\boldsymbol{u}(t)$ that constitute the input data, as shown in Fig. 4. Accordingly, $M = 2$ for our single-frequency case. Furthermore, all signals are real valued. One solution to the problem is sampling $\boldsymbol{u}(t)$ at around 40 MHz, which is approximately twice the bandwidth of interest. However, such an approach would merely double the cost of the receiver. Applying the hybrid approach, described by (6),(7), yields an efficient alternative. The resulting canceller structure is depicted in Fig. 4.

The choice of the update-rate $1/T$ depends mainly on the tracking requirements introduced by the application. For reference-based interference cancellation, the changes of correlation between disturber and ref-

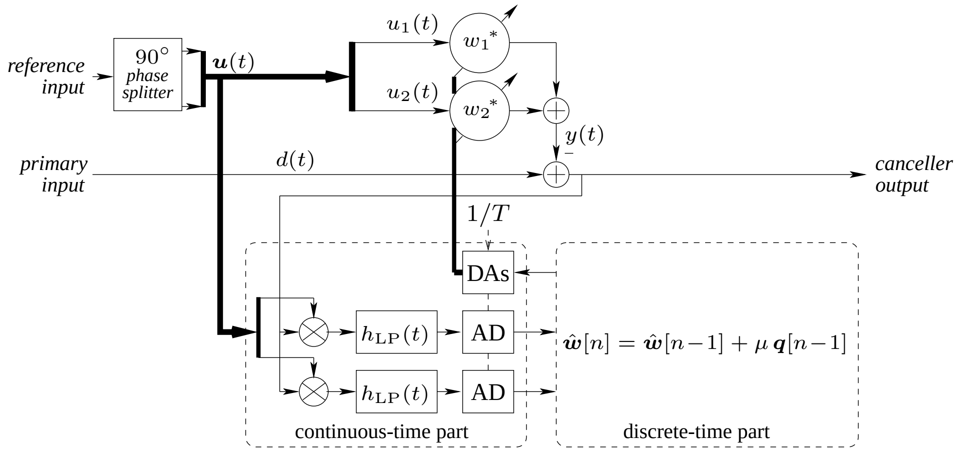

Figure 4: Block diagram of the narrowband interference canceller: the two-dimensional real-valued input vector constitutes a special case of the generic structure shown in Fig. 2.

erence have to be tracked. In comparison to the high-frequency disturbance, the correlation between disturber and reference varies very slowly, since neither the wire nor the disturbing amateur radio transmitter move quickly. Thus $1/T$ can be chosen to be orders of magnitude below the frequency of the disturber, which will be shown by the following simulation results.

4.3. Simulation Results and Discussion

Table 1 summarizes the parameters of a typical interference scenario. Fig. 5 shows both simulated and theoretical learning curve of the algorithm adapting the weights in order to cancel the interference appearing at $t = 0$. The graphs display the two performance measures of interest. The resulting RFI suppression in steady-state exceeds 20 dB, which is sufficient to avoid an AD converter overload and leaves several dB margin for imperfections in implementation, especially regarding the analog components. Convergence in the mean square is reached after approximately 150 iterations. To achieve a convergence time of 1 ms, which is

signal:	Gaussian, white within the bandwidth, zero-mean, power: -20 dBm, bandwidth: $0 \ldots 20$ MHz
interference:	Gaussian, white within the bandwidth, zero-mean, power: 30 dBm at CM, 0 dBm at DM, bandwidth: 100 kHz centered at 3.5 MHz
step size μ:	0.2
update-rate $1/T$:	50 kHz, 100 kHz, 500 kHz
lowpass filter:	characteristic: elliptic, order: 4, bandwidth: $1/(2T)$

Table 1: Summary of simulation parameters.

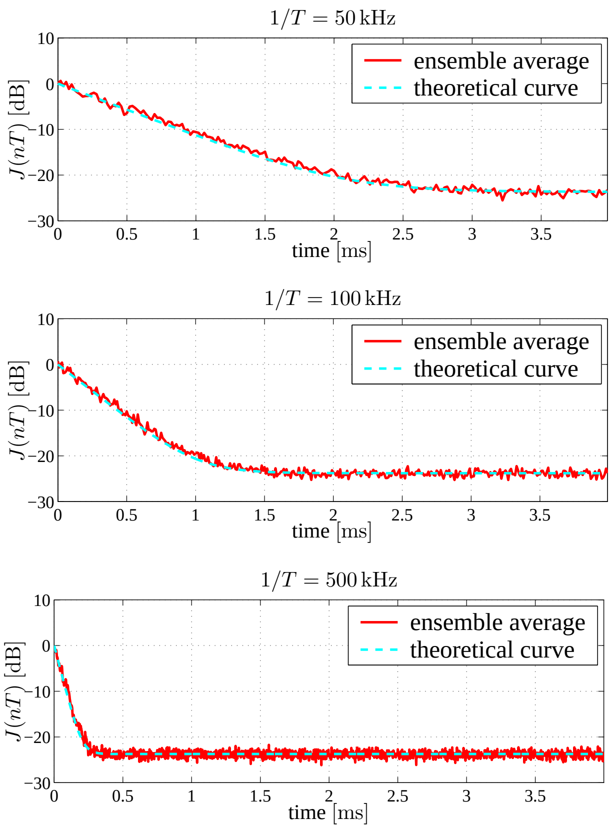

Figure 5: The learning curve (mean square error $J(n)$ versus time), shown for three different update-rates ($1/T = 50\,\mathrm{kHz}, 100\,\mathrm{kHz}, 500\,\mathrm{kHz}$), corresponds to the RFI suppression versus time for our interference cancellation example (ensemble size 100).

sufficient to combat intermittent amateur radio interference, an update rate of $1/T = 150\,\mathrm{kHz}$ is required. Compared to a sampling rate of $40\,\mathrm{MHz}$ required by the standard solution, the hybrid scheme allows to operate the AD converters at an approximately 250 times lower rate.

A drawback of this approach is that variations of the power levels of both CM and DM signals, without adapting the step size, can drastically worsen the convergence behaviour. A normalized LMS algorithm, which requires an additional AD converter, solves this problem. The structure of the normalized LMS version is identical to the corresponding RLS implementation [14] of this interference canceller.

5. CONCLUSIONS

In this paper we split the LMS algorithm, applied to adapt the continuous-time multiple-input linear combiner, into an analog and a digital part. We demonstrate the feasibility of the hybrid approach for high-frequency input data. The sampling rate of the AD converters is reduced to the update-rate of the algorithm, which may be orders of magnitude lower than the frequency of the processed signals.

Applying this approach to an interference cancellation problem occurring in wireline communications we show that the sampling rate of the AD converters can be reduced by a factor of 250. To summarize, this approach allows reduction of complexity in the analog domain at the cost of adaptation speed, which is determined by the application.

6. ACKNOWLEDGMENT

We are grateful to Maja Lončar for her careful proof-reading of the paper.

7. REFERENCES

[1] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, ISBN 0-13-322760-X, third edition, 1996.

[2] F. J. Kub and E. W. Justh, "Analog CMOS Implementation of High Frequency Least-Mean Square Error Learning Circuit," *IEEE J. Solid-State Circuits*, vol. 30, no. 12, pp. 1391–1398, Dec. 1995.

[3] T. Linder, H. Zojer, and B. Seger, "Fully Analogue LMS Adaptive Notch Filter in BiCMOS Technology," *IEEE J. Solid-State Circuits*, vol. 31, no. 1, pp. 81–89, Jan. 1996.

[4] H. Lev-Ari, T. Kailath, and J. M. Cioffi, "Adaptive Recursive-Least-Squares Lattice and Transversal Filters for Continuous-Time Signal Processing," *IEEE Trans. Circuits Syst. II*, vol. 29, no. 2, pp. 81–89, Feb. 1992.

[5] T. Magesacher, S. Haar, R. Zukunft, P. Ödling, T. Nordström, and P. O. Börjesson, "Splitting the Recursive Least-Squares Algorithm," in *Proc. 6th Int. Symp. on Signal Processing and its Applications ISSPA2001*, Kuala Lumpur, Malaysia, Aug. 2001, vol. I, pp. 319–322.

[6] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice Hall, Englewood Cliffs, 1985.

[7] B. Widrow, "Adaptive Noise Cancelling: Principles and Applications," *Proceedings of the IEEE*, vol. 63, pp. 1692–1716, Dec. 1975.

[8] P. Ödling, P. O. Börjesson, T. Magesacher, and T. Nordström, "An Approach to Analog Mitigation of RFI," *IEEE J. Select. Areas Commun.*, vol. 20, no. 5, pp. 974–986, June 2002.

[9] ITU-T, "Measuring arrangements to assess the degree of unbalance about earth," *ITU-T Recommendation O.9*, Mar. 1999.

[10] ITU-T, "Transmission Aspects of Unbalance about Earth," *ITU-T Recommendation G.117*, Feb. 1996.

[11] T. Magesacher, W. Henkel, T. Nordström, P. Ödling, and P. O. Börjesson, "On the Correlation between Common-Mode and Differential-Mode Signals," *Temporary Document TD 45, 013t45, ETSI STC TM6*, Sept. 2001.

[12] K. T. Foster and J. W. Cook, "The Radio Frequency Interference (RFI) environment for very high-rate transmission over metallic access wire-pairs," *ANSI Contribution T1E1.4/95-020*, 1995.

[13] L. de Clercq, M. Peeters, S. Schelstraete, and T. Pollet, "Mitigation of Radio Interference in xDSL Transmission," *IEEE Commun. Mag.*, vol. 38, no. 3, pp. 168–173, Mar. 2000.

[14] T. Magesacher, P. Ödling, T. Nordström, T. Lundberg, M. Isaksson, and P. O. Börjesson, "An Adaptive Mixed-Signal Narrowband Interference Canceller for Wireline Transmission Systems," in *Proc. IEEE Int. Symp. Circuits and Systems*, Sydney, Australia, May 2001, vol. IV, pp. 450–453.