Sorting Bits into RS Symbols According to their Reliability

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Abstract — **This paper discusses the advantages that can be expected from sorting bits into groups with similar reliability and gathering them into the symbols of a Reed-Solomon code.**

I. Motivation

In ADSL, a so-called 'tone ordering' has been provided to route bits from high-SNR tones through the interleaved channel, since those are more vulnerable to unexpected non-stationary noise. Gathering bits with equal reliability into common RS symbols would be a further step.

II. Results with two different bit error probabilities

We assume two different bit error rates p_i, $i = 1, 2$ and compare the resulting bit error rates after the decoding of the RS code with sorting with the error rate obtained without sorting.

The symbol-error rates of RS symbols of length m are

$$p_{si} = 1 - (1 - p_i)^m \quad \text{or} \quad p_{sm} = 1 - \prod_{i=1}^{2} (1 - p_i)^{m/2} , \qquad (1)$$

if the symbols are either filled with bits ordered according to their reliability or equally filled with bits of all reliability classes, respectively.

On the basis of these symbol error probabilities, we approximate the bit error probabilities after the decoding as

$$p_{bm,dec} \approx \sum_{j=t+1}^{N} \frac{j}{N} \cdot \frac{p_{bm}}{p_{sm}} \cdot \binom{N}{j} p_{sm}^{j} (1 - p_{sm})^{N-j} , \qquad (2)$$

$$p_{b,dec} \approx \sum_{j=t+1}^{N} \sum_{j_1=0}^{\min(j,N/2)} \binom{N/2}{j_1} p_{s1}^{j_1} (1 - p_{s1})^{N/2-j_1} \cdot$$

$$\cdot \binom{N/2}{j - j_1} p_{s2}^{j-j_1} (1 - p_{s2})^{N/2-j+j_1} \left(j_1 \frac{p_{b1}}{p_{s1}} + (j - j_1) \frac{p_{b2}}{p_{s2}} \right) / N , \qquad (3)$$

where we assume that the symbols of uncorrected received words have the same average bit error probability as the ones before decoding.

Figure 1 shows the ratio $p_{b,dec}/p_{bm,dec}$ with two different bit error rates p_1 and p_2. Although the number of bits was chosen to be equal, which is not optimum with respect to the code parameters, we still see a clear advantage for sorting when the two bit error rates are different.

We thus conclude that sorting should be applied for multicarrier modulation in combination with 'tone ordering' such that the bits from carriers with the same bit load are gathered in RS symbols.

Along these lines we also studied the bit-sorting for the different bits of Gray-coded QAM.

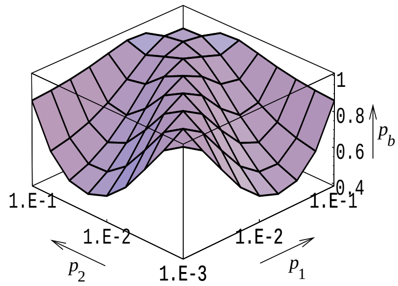

Figure 1: BERs after decoding of a (40,24) RS code over GF(2^6) as a function of the BERs before dec. p_i, $i = 1, 2$

III. Bit-sorting for QAM?

Gray mapping is obtained recursively from a 4-QAM mapping by placing the previous smaller constellation into a quadrant and mirroring along the axis. Moving the points in the quadrants further apart leads to so-called hierarchical modulation. With a spacing parameter $\alpha \geq 1$, we obtain, *e.g.*, four different bit error rates for the bits of a 256-QAM. Inside the quadrants, the differences are due to the numbers of nearest neighbors which are proportional to powers of 2 for Gray-coded mapping:

$$p_i = \frac{2 \cdot 2^{i-1}}{16} \cdot \begin{cases} p_\alpha , & i = 1 \\ p_0 , & i \geq 2 \end{cases} , \qquad (4)$$

$$p_\alpha = 1/2 \cdot \operatorname{erfc}(\alpha \sqrt{\mathrm{SNR}_0}) , \; p_0 = 1/2 \cdot \operatorname{erfc}(\sqrt{\mathrm{SNR}_0}) . \qquad (5)$$

SNR_0 is the SNR with respect to the distance of nearest points in a QAM. Extending (3) to four different bit error rates, we are able to compute the corresponding bit error performance after decoding. In Fig. 2 we see that, unfortunately, we do not gain much (*cf.*, $P_{b,dec}$ and $P_{bm,dec}$) by sorting the bits according to their error probabilities. The differences in bit error rates have to be more pronounced than those provided by Gray coding, even if some bits are more protected by an additional spacing in a hierarchical scheme. Sorting bits into separate codewords rather than just symbols does yield noticeable performance differences as demonstrated by curves 1-4 in Fig. 2.

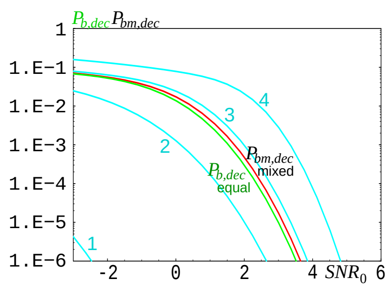

Figure 2: BERs after decoding of a (40,24) RS code over GF(2^6) of Gray-coded 256-QAM ($\alpha = 1.5$)